TMUA & MAT 好题集
(until 2022)

From ECFDPB
To OXBRIDGE-ers & IC-ers

微积分

2016TMUA-paper1-5

5 What is the total area enclosed between the curve $y = x^2 - 1$, the x-axis and the lines $x = -2$ and $x = 2$?

A $\frac{4}{3}$

B $\frac{8}{3}$

C 4

D $\frac{16}{3}$

E 12

F 16

2018TMUA-paper1-12

12 A curve has equation $y = \text{f}(x)$, where

$$\text{f}(x) = x(x-p)(x-q)(r-x)$$

with $0 < p < q < r$.

You are given that:

$$\int_0^r \text{f}(x)\,\text{d}x = 0$$

$$\int_0^q \text{f}(x)\,\text{d}x = -2$$

$$\int_p^r \text{f}(x)\,\text{d}x = -3$$

What is the total area enclosed by the curve and the x-axis for $0 \le x \le r$?

A 0

B 1

C 4

D 5

E 6

F 10

2010MAT-I

I. For a positive number a, let

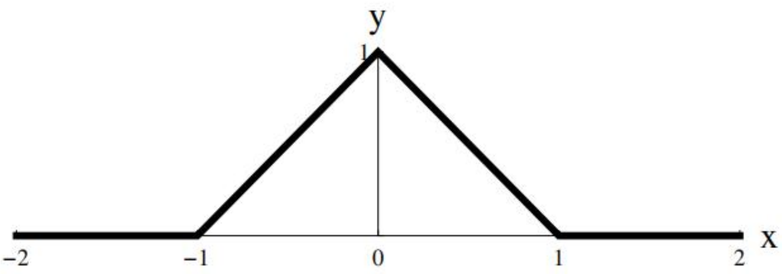
$$I(a) = \int_0^a \left(4 - 2^{x^2}\right) \mathrm{d}x.$$

Then $\mathrm{d}I/\mathrm{d}a = 0$ when a equals

(a) $\frac{1+\sqrt{5}}{2}$, (b) $\sqrt{2}$, (c) $\frac{\sqrt{5}-1}{2}$, (d) 1.

2011MAT-G

G. A graph of the function $y = f(x)$ is sketched on the axes below:



The value of $\int_{-1}^{1} f(x^2-1)\,\mathrm{d}x$ equals

(a) $\frac{1}{4}$, (b) $\frac{1}{3}$, (c) $\frac{3}{5}$, (d) $\frac{2}{3}$.

2013MAT-C

C. The functions f, g and h are related by

$$f'(x) = g(x+1), \qquad g'(x) = h(x-1).$$

It follows that $f''(2x)$ equals

(a) $h(2x+1)$; (b) $2h'(2x)$; (c) $h(2x)$; (d) $4h(2x)$.

2013MAT-E

E. The expression

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[(2x-1)^4(1-x)^5\right] + \frac{\mathrm{d}}{\mathrm{d}x}\left[(2x+1)^4\left(3x^2-2\right)^2\right]$$

is a polynomial of degree

(a) 9; (b) 8; (c) 7; (d) less than 7.

2014MAT-J

J. For all real numbers x, the function $f(x)$ satisfies

$$6 + f(x) = 2f(-x) + 3x^2 \left(\int_{-1}^{1} f(t)\,\mathrm{d}t \right).$$

It follows that $\int_{-1}^{1} f(x)\,\mathrm{d}x$ equals

(a) 4, (b) 6, (c) 11, (d) $\frac{27}{2}$, (e) 23.

1996MAT-H

(h) The derivative of the function $y = (\mathrm{e}^{\cos(5x)})^2$ is:

(i) $-5\sin(5x)(\mathrm{e}^{\cos(5x)})^2$; (ii) $-20\sin(5x)\cos(5x)(\mathrm{e}^{\cos(5x)})^2$;

(iii) $-10\sin(5x)(\mathrm{e}^{\cos(5x)})^2$; (iv) $-10\sin(5x)\cos(5x)(\mathrm{e}^{\cos(5x)})^2$.

2000MAT-G

G. The derivative of $xe^{-x^2}\cos\left(\frac{1}{x}\right)$ is

(a) $-\frac{1}{x}e^{-x^2}\sin\left(\frac{1}{x}\right) - 2x^2e^{-x^2}\cos\left(\frac{1}{x}\right) + e^{-x^2}\cos\left(\frac{1}{x}\right)$

(b) $\frac{1}{x}e^{-x^2}\sin\left(\frac{1}{x}\right) - 2x^2e^{-x^2}\cos\left(\frac{1}{x}\right) + e^{-x^2}\cos\left(\frac{1}{x}\right)$

(c) $\frac{1}{x}e^{-x^2}\sin\left(\frac{1}{x}\right) + 2x^2e^{-x^2}\cos\left(\frac{1}{x}\right) + e^{-x^2}\cos\left(\frac{1}{x}\right)$

(d) $\frac{1}{x}e^{-x^2}\cos\left(\frac{1}{x}\right) - 2x^2e^{-x^2}\cos\left(\frac{1}{x}\right) + e^{-x^2}\cos\left(\frac{1}{x}\right)$.

2021MAT-EXTRA-I

I. Given that there are positive real numbers a, b, c that satisfy

$$\int_a^b \log_c\left(\sin^4 x \tan^2 x\right)\,\mathrm{d}x = 1 \quad \text{and} \quad \int_a^b \log_c\left(\sin^2 x \cos^2 x\right)\,\mathrm{d}x = 3,$$

it follows that the value of

$$\int_a^b \log_c\left(\sin^4 x \cos^2 x\right)\,\mathrm{d}x$$

must be equal to

(a) 4,

(b) 5,

(c) 6,

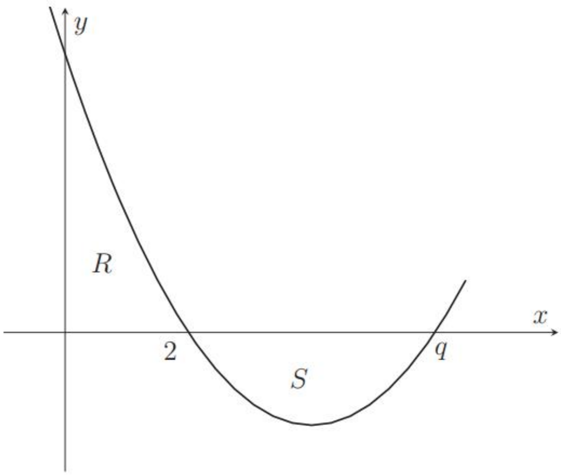
(d) 7,

(e) 8.

[Note that $\sin^4 x$ means $(\sin x)^4$.]

2020TMUA-paper1-11

11 The quadratic function shown passes through $(2, 0)$ and $(q, 0)$, where $q > 2$.



What is the value of q such that the area of region R equals the area of region S?

A $\sqrt{6}$

B 3

C $\frac{18}{5}$

D 4

E 6

F $\frac{33}{5}$

2022MAT-EXTRA-G

G. Given that $y = f(x)$ is a solution to $\dfrac{\mathrm{d}y}{\mathrm{d}x} = y^{1/4}$, it follows that one of the following functions is a solution to $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y^{1/4}$. Which one?

(a) $y = (2)^{-4} f(x)$,

(b) $y = (2)^{3} f(x)$,

(c) $y = (2)^{4/3} f(x)$,

(d) $y = (2)^{-3} f(x)$,

(e) $y = (2)^{4} f(x)$.

2021TMUA-paper1-7

7 The function f is such that $f(0) = 0$, and $xf(x) > 0$ for all non-zero values of x.

It is given that

$$\int_{-2}^{2} f(x)\, dx = 4$$

and

$$\int_{-2}^{2} |f(x)|\, dx = 8$$

Evaluate

$$\int_{-2}^{0} f(|x|)\, dx$$

A -8

B -6

C -4

D -2

E 2

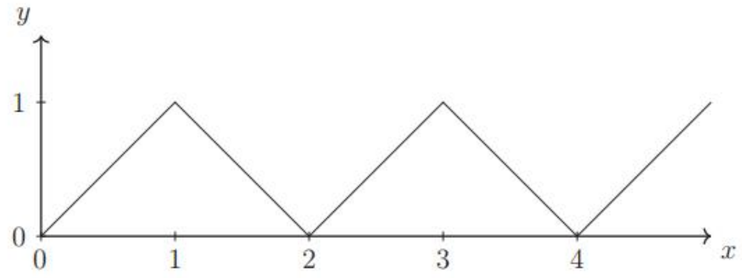
F 4

G 6

H 8

2021TMUA-paper1-15

15 The diagram shows the graph of $y = \mathrm{f}(x)$.



The graph consists of alternating straight-line segments of gradient 1 and -1 and continues in this way for all values of x.

The function g is defined as

$$\mathrm{g}(x) = \sum_{r=1}^{10} f\left(2^{r-1}x\right)$$

Find the value of

$$\int_0^1 \mathrm{g}(x)\,\mathrm{d}x$$

A $\frac{1023}{1024}$

B $\frac{1023}{512}$

C 5

D 10

E $\frac{55}{2}$

F 55

画图

2017TMUA-paper1-18

18 The graph of $y = \log_{10}x$ is translated in the positive y-direction by 2 units.

This translation is equivalent to a stretch of factor k parallel to the x-axis.

What is the value of k?

A 0.01

B $\log_{10} 2$

C 0.5

D 2

E $\log_2 10$

F 100

2017TMUA-paper2-10

10 $f(x)$ is a function defined for all real values of x.

Which one of the following is a **sufficient** condition for $\int_1^3 f(x)\,dx = 0$?

A $f(2) = 0$

B $f(1) = f(3) = 0$

C $f(-x) = -f(x)$ for all x

D $f(x+2) = -f(2-x)$ for all x

E $f(x-2) = -f(2-x)$ for all x

2018TMUA-paper2-15

15 It is given that $f(x) = x^3 + 3qx^2 + 2$, where q is a real constant.

The equation $f(x) = 0$ has 3 distinct real roots.

Which of the following statements is/are **necessarily** true?

I The equation $f(x) + 1 = 0$ has 3 distinct real roots.

II The equation $f(x + 1) = 0$ has 3 distinct real roots.

III The equation $f(-x) - 1 = 0$ has 3 distinct real roots.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

2009MAT-F

F. The equation in x

$$3x^4 - 16x^3 + 18x^2 + k = 0$$

has four real solutions
(a) when $-27 < k < 5$;
(b) when $5 < k < 27$;
(c) when $-27 < k < -5$;
(d) when $-5 < k < 0$.

2010MAT-J

J. Let a, b, c be positive numbers. There are *finitely* many *positive whole* numbers x, y which satisfy the inequality

$$a^x > c\,b^y$$

if

(a) $a > 1$ or $b < 1$.
(b) $a < 1$ or $b < 1$.
(c) $a < 1$ and $b < 1$.
(d) $a < 1$ and $b > 1$.

2019TMUA-paper2-20

20 When the graph of the function $y = f(x)$, defined on the real numbers, is reflected in the y-axis and then translated by 2 units in the negative x-direction, the result is the graph of the function $y = g(x)$.

When the graph of the same function $y = f(x)$ is translated by 2 units in the negative x-direction and then reflected in the y-axis, the result is the graph of the function $y = h(x)$.

Which one of the following conditions on $y = f(x)$ is **necessary and sufficient** for the functions $g(x)$ and $h(x)$ to be identical?

A $f(x) = f(x + 2)$ for all x

B $f(x) = f(x + 4)$ for all x

C $f(x) = f(x + 8)$ for all x

D $f(x) = f(-x)$ for all x

E $f(x) = f(2 - x)$ for all x

F $f(x) = f(4 - x)$ for all x

G $f(x) = f(8 - x)$ for all x

2015MAT-C

C. Which of the following are true for all real values of x? All arguments are in radians.

I $\sin\left(\frac{\pi}{2} + x\right) = \cos\left(\frac{\pi}{2} - x\right)$

II $2 + 2\sin(x) - \cos^2(x) \geqslant 0$

III $\sin\left(x + \frac{3\pi}{2}\right) = \cos(\pi - x)$

IV $\sin(x)\cos(x) \leqslant \frac{1}{4}$

(a) **I** and **II**, (b) **I** and **III**, (c) **II** and **III**,
(d) **III** and **IV**, (e) **II** and **IV**.

2022TMUA-paper1-18

18 It is given that

$$f(x) = x^2(x-1)^2(x-2)$$

$$g(x) = -p(x-q)^2(x-r)^2$$

where p, q and r are positive and $q < r$

Find the set of values of q and r that guarantees the greatest number of distinct real solutions of the equation $f(x) = g(x)$ for all p.

A $q < 1$ and $r < 1$

B $q < 1$ and $1 < r < 2$

C $q < 1$ and $r > 2$

D $1 < q < 2$ and $1 < r < 2$

E $1 < q < 2$ and $r > 2$

F $q > 2$ and $r > 2$

2020TMUA-paper1-10

10 The following sequence of transformations is applied to the curve $y = 4x^2$

1. Translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$
2. Reflection in the x-axis
3. Stretch parallel to the x-axis with scale factor 2

What is the equation of the resulting curve?

A $y = -x^2 + 12x - 31$

B $y = -x^2 + 12x - 41$

C $y = x^2 + 12x + 31$

D $y = x^2 + 12x + 41$

E $y = -16x^2 + 48x - 31$

F $y = -16x^2 + 48x - 41$

G $y = 16x^2 - 48x + 31$

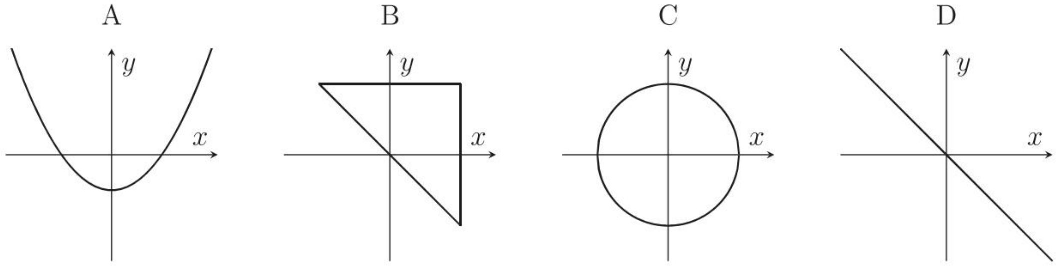
H $y = 16x^2 - 48x + 41$

2018MAT-J

J. Which of the following could be the sketch of a curve

$$p(x) + p(y) = 0$$

for some polynomial p?



(a) A and D, but not B or C;
(b) A and B, but not C or D;
(c) C and D, but not A or B;
(d) A, C and D, but not B;
(e) A, B and C, but not D.

2022MAT-J

J. The real numbers m and c are such that the equation

$$x^2 + (mx + c)^2 = 1$$

has a repeated root x, and also the equation

$$(x - 3)^2 + (mx + c - 1)^2 = 1$$

has a repeated root x (which is not necessarily the same value of x as the root of the first equation). How many possibilities are there for the line $y = mx + c$?

(a) 0, (b) 1, (c) 2, (d) 3, (e) 4.

2021TMUA-paper2-16

16 p and q are real numbers, and the equation

$$x\,|x| = px + q$$

has exactly k distinct real solutions for x.

Which one of the following is the complete list of possible values for k?

A 0, 1, 2

B 0, 1, 2, 3

C 0, 1, 2, 3, 4

D 0, 2, 4

E 1, 2, 3

F 1, 2, 3, 4

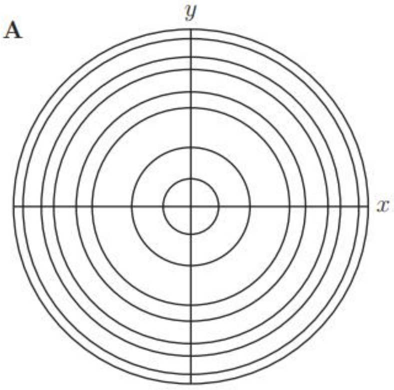
2021TMUA-paper1-17

17 Which of the following sketches shows the graph of

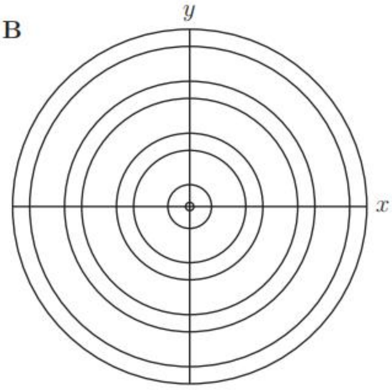
$$\sin(x^2 + y^2) = \frac{1}{2}$$

where $x^2 + y^2 \leq 8\pi$?

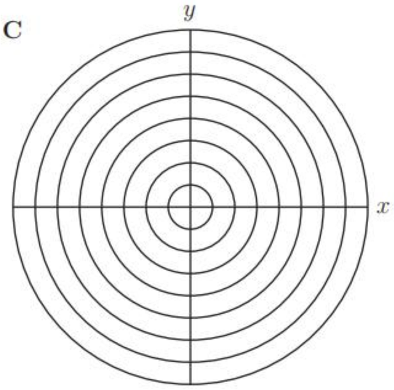
A



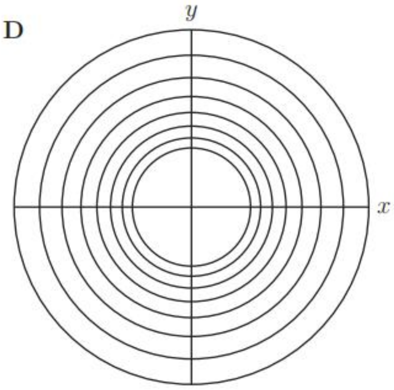
B



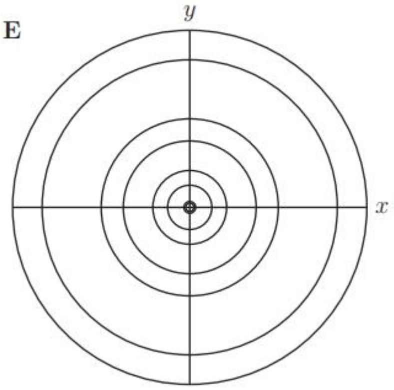
C



D



E



2021TMUA-paper1-18

18 The curve with equation

$$x = y^2 - 6y + 11$$

is rotated 90° clockwise about the point P to give the curve C.

P has x-coordinate -2 and y-coordinate 3.

What is the equation of C ?

A $y = -x^2 - 4x - 3$

B $y = -x^2 - 4x - 5$

C $y = -x^2 - 6x - 7$

D $y = -x^2 - 6x - 11$

E $y = \ \ x^2 - 4x + 5$

F $y = \ \ x^2 + 4x + 3$

G $y = \ \ x^2 - 6x + 11$

H $y = \ \ x^2 + 6x + 7$

2021TMUA-paper1-20

20 Find the length of the curve with equation

$$2\log_{10}(x - y) = \log_{10}(2 - 2x) + \log_{10}(y + 5)$$

A 5

B 10

C 15

D 3π

E 9π

F 12π

2020MAT-EXTRA-I

I. An equilateral triangle is drawn in the xy-plane. Two of its vertices are at $(0, 0)$ and $(1000, 0)$. The number of points (x, y) *inside* the triangle, where x and y are both whole numbers, equals

(a) 866,025, (b) 866,026, (c) 866,027, (d) 432,512, (e) 432,513.

[Note that $\sqrt{3} = 1.7321$ to 4 decimal places.]

2020MAT-EXTRA-J

J. Let R be the region where all four of the following inequalities hold

$$x^2 < 2 + y, \quad x^2 < 2 - y, \quad y^2 < 2 + x, \quad y^2 < 2 - x.$$

What is the area of R?

(a) 0, (b) $\frac{28}{3}$, (c) $4 + 2\pi$, (d) $\frac{4}{3}\left(8\sqrt{2} - 7\right)$, (e) infinite.

2021MAT-EXTRA-J

J. There is a straight line that is normal to the curve $y = x^3 - kx$ at two different points if and only if

(a) $k \geq \sqrt{3}$,

(b) $k^2 \geq 3$,

(c) $k^2 \geq 1$,

(d) $k \geq 1$,

(e) $k \geq \sqrt{3}$ or $k \leq -1$.

2023MAT-EXTRA-I

I. Consider the nine lines $y = 2x + 1$, $y = 2x + 2$, $\ldots$, $y = 2x + 9$ and the seven lines $y = -x + 1$, $y = -x + 2$, $\ldots$, $y = -x + 7$.

How many distinct points are there at which the line $y = 1 - 10x$ crosses one or more of the other lines?

(a) 12

(b) 13

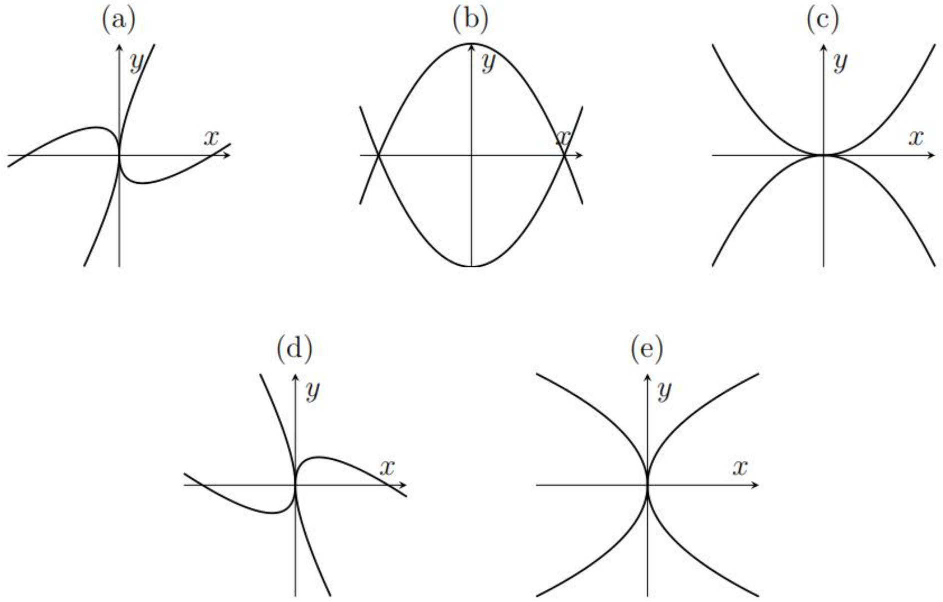
(c) 14

(d) 15

(e) 16

2023MAT-EXTRA-J

J. Which of the following is the graph of $y(y^3 + 4y^2x + 4x^3) = x^2(1 - x^2 - 6y^2)$?



SPECIMEN_TMUA_paper1-10

10. The curve $y = \cos x$ is reflected in the line $y = 1$ and the resulting curve is then translated by $\frac{\pi}{4}$ units in the positive x-direction. The equation of this new curve is

A $y = 2 + \cos\left(x + \frac{\pi}{4}\right)$

B $y = 2 + \cos\left(x - \frac{\pi}{4}\right)$

C $y = 2 - \cos\left(x + \frac{\pi}{4}\right)$

D $y = 2 - \cos\left(x - \frac{\pi}{4}\right)$

SPECIMEN_TMUA_paper2-1

1. The radius of the circle $2x^2 + 2y^2 - 8x + 12y + 15 = 0$ is

A $\sqrt{\frac{5}{2}}$

B $\sqrt{\frac{11}{2}}$

C $\sqrt{\frac{41}{2}}$

D $\sqrt{37}$

E $\sqrt{67}$

多项式

2018TMUA-paper1-8

8 The sum to infinity of a geometric progression is 6.

The sum to infinity of the squares of each term in the progression is 12.

Find the sum to infinity of the cubes of each term in the progression.

A 8

B 18

C 24

D $\frac{216}{7}$

E 72

F 216

2018TMUA-paper2-19

19 Three **real** numbers x, y and z satisfy $x > y > z > 1$.

Which one of the following statements **must** be true?

A $\frac{2^{z+1}}{2^x} > \frac{2^x + 2^z}{2^y}$

B $2 > \frac{3^x + 3^z}{3^y}$

C $\frac{2 \times 5^x}{5^z} > \frac{5^x + 5^z}{5^y}$

D $2 < \frac{7^x + 7^z}{7^y}$

2008MAT-C

C. The simultaneous equations in x, y,

$$\begin{aligned}(\cos\theta)\,x - (\sin\theta)\,y &= 2\\(\sin\theta)\,x + (\cos\theta)\,y &= 1\end{aligned}$$

are solvable

(a) for all values of θ in the range $0 \leqslant \theta < 2\pi$;
(b) except for one value of θ in the range $0 \leqslant \theta < 2\pi$;
(c) except for two values of θ in the range $0 \leqslant \theta < 2\pi$;
(d) except for three values of θ in the range $0 \leqslant \theta < 2\pi$.

2019TMUA-paper1-14

14 x satisfies the simultaneous equations

$$\sin 2x + \sqrt{3}\cos 2x = -1$$

and

$$\sqrt{3}\sin 2x - \cos 2x = \sqrt{3}$$

where $0° \leq x \leq 360°$.

Find the sum of the possible values of x.

A $210°$

B $330°$

C $390°$

D $660°$

E $780°$

F $930°$

2019TMUA-paper1-15

15 Find the real non-zero solution to the equation

$$\frac{2^{(9^x)}}{8^{(3^x)}} = \frac{1}{4}$$

A $\log_3 2$

B $2\log_3 2$

C 1

D 2

E $\log_2 3$

F $2\log_2 3$

2009_SPECIMEN_MAT_2-H

H. The equation

$$\left(x^2+1\right)^{10} = 2x - x^2 - 2$$

(a) has $x = 2$ as a solution;
(b) has no real solutions;
(c) has an odd number of real solutions;
(d) has twenty real solutions.

2009_SPECIMEN_MAT_2-I

I. .Observe that $2^3 = 8$, $2^5 = 32$, $3^2 = 9$ and $3^3 = 27$. From these facts, we can deduce that $\log_2 3$, the logarithm of 3 to base 2, is

(a) between $1\frac{1}{3}$ and $1\frac{1}{2}$;
(b) between $1\frac{1}{2}$ and $1\frac{2}{3}$;
(c) between $1\frac{2}{3}$ and 2;
(d) between 2 and 3.

2008MAT-J

J. In the range $0 \leqslant x < 2\pi$ the equation

$$(3 + \cos x)^2 = 4 - 2\sin^8 x$$

has

(a) 0 solutions, (b) 1 solution, (c) 2 solutions, (d) 3 solutions.

2009MAT-C

C. Given a real constant c, the equation

$$x^4 = (x - c)^2$$

has four real solutions (including possible repeated roots) for

(a) $c \leqslant \frac{1}{4}$, (b) $-\frac{1}{4} \leqslant c \leqslant \frac{1}{4}$, (c) $c \leqslant -\frac{1}{4}$, (d) all values of c.

2009MAT-I

I. The polynomial

$$n^2 x^{2n+3} - 25nx^{n+1} + 150x^7$$

has $x^2 - 1$ as a factor
(a) for no values of n;
(b) for $n = 10$ only;
(c) for $n = 15$ only;
(d) for $n = 10$ and $n = 15$ only.

2010MAT-C

C. In the range $0 \leqslant x < 2\pi$, the equation

$$\sin^2 x + 3\sin x \cos x + 2\cos^2 x = 0$$

has

(a) 1 solution, (b) 2 solutions, (c) 3 solutions, (d) 4 solutions.

2010MAT-H

H. Given a positive integer n and a real number k, consider the following equation in x,

$$(x-1)(x-2)(x-3)\times\cdots\times(x-n)=k.$$

Which of the following statements about this equation is true?

(a) If $n=3$, then the equation has no real solution x for some values of k.
(b) If n is even, then the equation has a real solution x for any given value of k.
(c) If $k \geqslant 0$ then the equation has (at least) one real solution x.
(d) The equation never has a repeated solution x for any given values of k and n.

2011MAT-F

F. Given θ in the range $0 \leqslant \theta < \pi$, the equation

$$x^2+y^2+4x\cos\theta+8y\sin\theta+10=0$$

represents a circle for

(a) $0<\theta<\dfrac{\pi}{3}$, (b) $\dfrac{\pi}{4}<\theta<\dfrac{3\pi}{4}$, (c) $0<\theta<\dfrac{\pi}{2}$, (d) all values of θ.

2011MAT-I

I. In the range $0 \leqslant x < 2\pi$ the equation

$$\sin^8 x+\cos^6 x=1$$

has

(a) 3 solutions , (b) 4 solutions, (c) 6 solutions, (d) 8 solutions.

2012MAT-G

G. There are *positive* real numbers x and y which solve the equations

$$2x+ky=4, \qquad x+y=k$$

for

(a) all values of k; (b) no values of k; (c) $k=2$ only; (d) only $k>-2$.

2013MAT-F

F. Three *positive* numbers a, b, c satisfy

$$\log_b a=2, \qquad \log_b(c-3)=3, \qquad \log_a(c+5)=2.$$

This information

(a) specifies a uniquely.
(b) is satisfied by two values of a.
(c) is satisfied by infinitely many values of a.
(d) is contradictory.

2013MAT-G

G. Let $n \geqslant 2$ be an integer and $p_n(x)$ be the polynomial

$$p_n(x) = (x-1) + (x-2) + \cdots + (x-n).$$

What is the remainder when $p_n(x)$ is divided by $p_{n-1}(x)$?

(a) $\dfrac{n}{2}$; (b) $\dfrac{n+1}{2}$; (c) $\dfrac{n^2+n}{2}$; (d) $\dfrac{-n}{2}$.

2014MAT-E

E. As x varies over the real numbers, the largest value taken by the function

$$\left(4\sin^2 x + 4\cos x + 1\right)^2$$

equals

(a) $17+12\sqrt{2}$, (b) 36, (c) $48\sqrt{2}$, (d) $64-12\sqrt{3}$, (e) 81.

2016MAT-I

I. Let a and b be positive real numbers. If $x^2 + y^2 \leqslant 1$ then the largest that $ax + by$ can equal is

(a) $\dfrac{1}{a}+\dfrac{1}{b}$, (b) $\max(a,b)$, (c) $\sqrt{a^2+b^2}$, (d) $a+b$, (e) a^2+ab+b^2.

2017MAT-I

I. Let $a, b, c > 0$ and $a \neq 1$. The equation

$$\log_b\left((b^x)^x\right) + \log_a\left(\frac{c^x}{b^x}\right) + \log_a\left(\frac{1}{b}\right)\log_a(c) = 0$$

has a repeated root when

(a) $b^2 = 4ac$ (b) $b = \dfrac{1}{a}$ (c) $c = \dfrac{b}{a}$ (d) $c = \dfrac{1}{b}$ (e) $a = b = c$.

2015MAT-B

B. Let

$$f(x) = (x+a)^n$$

where a is a real number and n is a positive whole number, and $n \geqslant 2$. If $y = f(x)$ and $y = f'(x)$ are plotted on the same axes, the number of intersections between $f(x)$ and $f'(x)$ will

(a) always be odd, (b) always be even, (c) depend on a but not n,
(d) depend on n but not a, (e) depend on both a and n.

1997MAT-K

(k) The simultaneous equations

$$\begin{aligned} x - 2y + 3z &= 1 \\ 2x + 3y - z &= 4 \\ 4x - y + 5z &= 6 \end{aligned}$$

have
(i) no solutions,
(ii) exactly one solution,
(iii) exactly three solutions,
(iv) infinitely many solutions.

1998MAT-D

(d) The simultaneous equations

$$2x + ay = b,$$
$$x + 3y = 4,$$

(i) when $a = 6$ and b is any value, do not have a solution,
(ii) when $b \neq 8$ and a is any value, have a solution,
(iii) have a solution for any values of a and b,
(iv) when either $a \neq 6$ or $b = 8$, have a solution,
(v) satisfy none of these.

2000MAT-E

E. The maximum gradient of the curve $y = x^4 - 4x^3 + 4x^2 + 2$ in the range $0 \leq x \leq 2\frac{1}{5}$ occurs when

(a) $x = 0$ (b) $x = 1 - \frac{1}{\sqrt{3}}$ (c) $x = 1 + \frac{1}{\sqrt{3}}$ (d) $x = 2\frac{1}{5}$.

2020TMUA-paper1-8

8 The function f is defined for all real x as

$$f(x) = (p - x)(x + 2)$$

Find the complete set of values of p for which the maximum value of $f(x)$ is less than 4.

A $-2 - 4\sqrt{2} < p < -2 + 4\sqrt{2}$

B $-2 - 2\sqrt{2} < p < -2 + 2\sqrt{2}$

C $-2\sqrt{5} < p < 2\sqrt{5}$

D $-6 < p < 2$

E $-4 < p < 0$

F $-2 < p < 2$

2020TMUA-paper1-9

9 The quadratic expression $x^2 - 14x + 9$ factorises as $(x - \alpha)(x - \beta)$, where α and β are positive real numbers.

Which quadratic expression can be factorised as $(x - \sqrt{\alpha})(x - \sqrt{\beta})$?

A $x^2 - \sqrt{10}x + 3$

B $x^2 - \sqrt{14}x + 3$

C $x^2 - \sqrt{20}x + 3$

D $x^2 - 178x + 81$

E $x^2 - 176x + 81$

F $x^2 + 196x + 81$

2020TMUA-paper1-17

17 Find the complete set of values of m in terms of c such that the graphs of $y = mx + c$ and $y = \sqrt{x}$ have two points of intersection.

A $0 < m < \frac{1}{4c}$

B $0 < m < 4c^2$

C $m > \frac{1}{4c}$

D $m < \frac{1}{4c}$

E $m > 4c^2$

F $m < 4c^2$

2020TMUA-paper1-19

19 Find the lowest positive integer for which $x^2 - 52x - 52$ is positive.

A 26

B 27

C 51

D 52

E 53

F 54

2020TMUA-paper2-18

18 In this question, $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = px^3 + qx^2 + rx + s$ are cubic polynomials.

If $f(x) - g(x) > 0$ for every real x, which of the following is/are **necessarily** true?

I $a > p$

II **if** $b = q$ **then** $c = r$

III $d > s$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

2002MAT-A

A. The number of solutions of the equation

$$x^3 + ax^2 - x - 2 = 0$$

for which $x > 0$ is

(a) 1 (b) 2 (c) 3 (d) dependent on the value of a.

2003MAT-A

A. Depending on the value of the constant d, the equation

$$dx^2 - (d-1)x + d = 0$$

may have two real solutions, one real solution or no real solutions. For how many values of d does it have *just one* real solution?

(a) for one value of d ;

(b) for two values of d ;

(c) for three values of d ;

(d) for infinitely many values of d .

2018MAT-G

G. The parabolas with equations $y = x^2 + c$ and $y^2 = x$ touch (that is, meet tangentially) at a single point. It follows that c equals

(a) $\dfrac{1}{2\sqrt{3}}$, (b) $\dfrac{3}{4\sqrt[3]{4}}$, (c) $\dfrac{-1}{2}$, (d) $\sqrt{5} - \sqrt{3}$, (e) $\sqrt{\dfrac{2}{3}}$.

2019MAT-F

F. In the interval $0 \leqslant x < 360°$, the equation

$$\sin^3 x + \cos^2 x = 0$$

has

(a) 0, (b) 1, (c) 2, (d) 3, (e) 4

solutions.

2019MAT-G

G. Let $a, b, c > 0$. The equations

$$\log_a b = c, \qquad \log_b a = c + \frac{3}{2}, \qquad \log_c a = b,$$

(a) specify a, b and c uniquely.
(b) specify c uniquely but have infinitely many solutions for a and b.
(c) specify c and a uniquely but have infinitely many solutions for b.
(d) specify a and b uniquely but have infinitely many solutions for c.
(e) have no solutions for a, b and c.

2020MAT-I

I. In the range $-90° < x < 90°$, how many values of x are there for which the sum to infinity

$$\frac{1}{\tan x} + \frac{1}{\tan^2 x} + \frac{1}{\tan^3 x} + \ldots$$

equals $\tan x$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4.

2004MAT-A

A. How many values of x satisfy the equation

$$\sin 2x + \sin^2 x = 1$$

in the range $0 \leq x < 2\pi$?

(a) 2 (b) 4 (c) 6 (d) 8

2021MAT-EXTRA-E

E. The polynomial equation $x^4 - (2k+1)x^2 + 2x + k^2 - 1 = 0$ has exactly four real solutions x if and only if

(a) $k > 1$,

(b) $k > -\frac{5}{4}$,

(c) $k > \frac{3}{4}$,

(d) $k < -\frac{5}{4}$ or $k > \frac{3}{4}$,

(e) $\frac{3}{4} < k < 1$ or $k > 1$.

2021TMUA-paper2-14

14 Consider the following simultaneous equations, where p is a real number:

$$p2^x + \log_2 y = 2$$

$$2^x + \log_2 y = 1$$

What is the complete range of p for which these simultaneous equations have a real solution (x, y)?

A $p < 1$

B $p \neq 1$

C $p > 1$

D $p < 1$ or $p > 2$

E $p \neq 1$ and $p < 2$

F $p > 1$ and $p < 2$

G $p > 2$

H All real values of p

2021TMUA-paper2-17

17 Consider the following functions defined for $x > 1$:

$$\mathrm{f}(x) = \log_2\left(\log_2 \sqrt{x}\right)$$

$$\mathrm{g}(x) = \log_2\left(\sqrt{\log_2 x}\right)$$

Which one of the following is true for **all** values of $x > 1$?

A $0 \leq \mathrm{f}(x) \leq \mathrm{g}(x)$ **or** $\mathrm{g}(x) \leq \mathrm{f}(x) \leq 0$

B $0 \leq \mathrm{g}(x) \leq \mathrm{f}(x)$ **or** $\mathrm{f}(x) \leq \mathrm{g}(x) \leq 0$

C $\frac{1}{2} \leq \mathrm{f}(x) \leq \mathrm{g}(x)$ **or** $\mathrm{g}(x) \leq \mathrm{f}(x) \leq \frac{1}{2}$

D $\frac{1}{2} \leq \mathrm{g}(x) \leq \mathrm{f}(x)$ **or** $\mathrm{f}(x) \leq \mathrm{g}(x) \leq \frac{1}{2}$

E $1 \leq \mathrm{f}(x) \leq \mathrm{g}(x)$ **or** $\mathrm{g}(x) \leq \mathrm{f}(x) \leq 1$

F $1 \leq \mathrm{g}(x) \leq \mathrm{f}(x)$ **or** $\mathrm{f}(x) \leq \mathrm{g}(x) \leq 1$

2021TMUA-paper2-19

19 The angle θ can take any of the values $1°$, $2°$, $3°$, ..., $359°$, $360°$.

For how many of these values of θ is it true that

$$\sin\theta\sqrt{1+\sin\theta}\sqrt{1-\sin\theta}+\cos\theta\sqrt{1+\cos\theta}\sqrt{1-\cos\theta}=0$$

A 0

B 1

C 2

D 4

E 93

F 182

G 271

H 360

2022MAT-EXTRA-C

C. For precisely which non-zero real values of x is it true that

$$x^2-3x+2<\frac{x-1}{x} \quad ?$$

(a) $x<1-\sqrt{2}$ or $x>1+\sqrt{2}$,

(b) $1-\sqrt{2}<x<0$ or $1<x<1+\sqrt{2}$,

(c) $1<x<1+\sqrt{2}$,

(d) $1-\sqrt{2}<x<1+\sqrt{2}$,

(e) $1-\sqrt{2}<x<0$.

2023MAT-EXTRA-H

H. Let $p(x)=2x^4-3x^3-5x^2+2x+2$. Given that the $y=mx$, with m a real number, crosses the curve $y=p(x)$ at four distinct points, let the x-coordinates of those points be x_1, x_2, x_3, and x_4. The product $x_1x_2x_3x_4$ is equal to

(a) 0

(b) 1

(c) 2

(d) 3

(e) Not enough information

SPECIMEN_TMUA_paper2-14

14. The graph of the polynomial function

$$y = ax^5 + bx^4 + cx^3 + dx^2 + ex + f,$$

is sketched, where a, b, c, d, e, and f are real constants with $a \neq 0$.

Which one of the following is **not** possible?

A The graph has two local minima and two local maxima.

B The graph has one local minimum and two local maxima.

C The graph has one local minimum and one local maximum.

D The graph has no local minima or local maxima.

SPECIMEN_TMUA_paper2-19

19. The positive real numbers a, b, and c are such that the equation

$$x^3 + ax^2 = bx + c$$

has three real roots, one positive and two negative.

Which one of the following correctly describes the real roots of the equation

$$x^3 + c = ax^2 + bx\ ?$$

A It has three real roots, one positive and two negative.

B It has three real roots, two positive and one negative.

C It has three real roots, but their signs differ depending on a, b, and c.

D It has exactly one real root, which is positive.

E It has exactly one real root, which is negative.

F It has exactly one real root, whose sign differs depending on a, b, and c.

G The number of real roots can be one or three, but the number of roots differs depending on a, b, and c.

几何

2018TMUA-paper1-19

19 A triangle ABC is to be drawn with $AB = 10\text{cm}$, $BC = 7\text{cm}$ and the angle at A equal to θ, where θ is a certain specified angle.

Of the two possible triangles that could be drawn, the larger triangle has three times the area of the smaller one.

What is the value of $\cos\theta$?

A $\frac{5}{7}$

B $\frac{151}{200}$

C $\frac{2\sqrt{2}}{5}$

D $\frac{\sqrt{17}}{5}$

E $\frac{\sqrt{51}}{8}$

F $\frac{\sqrt{34}}{8}$

2019TMUA-paper1-18

18 Find the shortest distance between the curve $y = x^2 + 4$ and the line $y = 2x - 2$.

A 2

B $\sqrt{5}$

C $\frac{6\sqrt{5}}{5}$

D 3

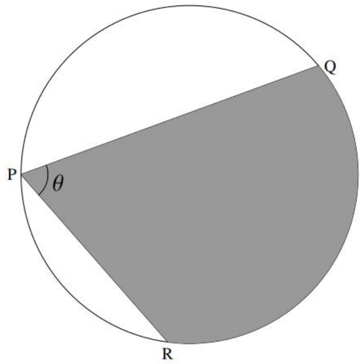
E $\frac{5\sqrt{5}}{3}$

F 5

G 6

2012MAT-J

J. If two chords QP and RP on a circle of radius 1 meet in an angle θ at P, for example as drawn in the diagram below,



then the largest possible area of the shaded region RPQ is

(a) $\theta\left(1+\cos\left(\frac{\theta}{2}\right)\right)$; (b) $\theta+\sin\theta$; (c) $\frac{\pi}{2}(1-\cos\theta)$; (d) θ.

2022TMUA-paper1-15

15 A rectangle is drawn in the region enclosed by the curves p and q, where

$$p(x) = 8 - 2x^2$$

$$q(x) = x^2 - 2$$

such that the sides of the rectangle are parallel to the x- and y-axes.

What is the maximum possible area of the rectangle?

A $\frac{26}{9}$

B $\frac{52}{9}$

C $\frac{4\sqrt{6}}{3}$

D $\frac{8\sqrt{6}}{3}$

E $4\sqrt{2}$

F $8\sqrt{2}$

G $\frac{20\sqrt{10}}{9}$

H $\frac{40\sqrt{10}}{9}$

2020TMUA-paper1-16

16 The circle C_1 has equation $(x+2)^2+(y-1)^2=3$

The circle C_2 has equation $(x-4)^2+(y-1)^2=3$

The straight line l is a tangent to both C_1 and C_2 and has positive gradient.

The acute angle between l and the x-axis is θ

Find the value of $\tan\theta$

A $\frac{1}{2}$

B 2

C $\frac{\sqrt{2}}{2}$

D $\sqrt{2}$

E $\frac{\sqrt{6}}{2}$

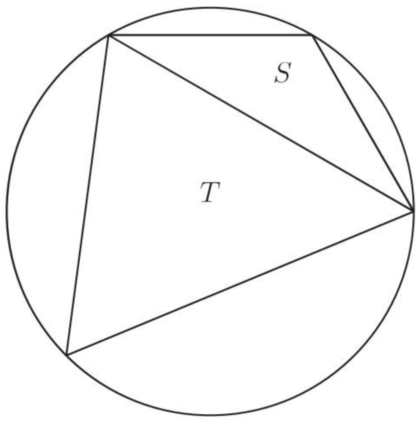
F $\frac{\sqrt{6}}{3}$

G $\frac{\sqrt{3}}{3}$

H $\sqrt{3}$

2018MAT-H

H. Two triangles S and T are inscribed in a circle, as shown in the diagram below.



The triangles have respective areas s and t and S is the smaller triangle so that $s < t$. The smallest value that

$$\frac{4s^2+t^2}{5st}$$

can equal is

(a) $\frac{2}{5}$, (b) $\frac{3}{5}$, (c) $\frac{4}{5}$, (d) 1, (e) $\frac{3}{2}$.

2021TMUA-paper1-18

18 A student chooses two distinct real numbers x and y with $0 < x < y < 1$.

The student then attempts to draw a triangle ABC with:

$$AB = 1$$
$$\sin A = x$$
$$\sin B = y$$

Which of the following statements is/are correct?

I For some choice of x and y, there is exactly **one** triangle the student could draw.

II For some choice of x and y, there are exactly **two** different triangles the student could draw.

III For some choice of x and y, there are exactly **three** different triangles the student could draw.

(Note that congruent triangles are considered to be the same.)

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

2021MAT-EXTRA-F

F. The point A has coordinates $(3, 4)$. The origin $(0, 0)$ and the point A both lie on the circumference of a circle $\mathcal{C}$. The diameter of $\mathcal{C}$ through A also meets $\mathcal{C}$ at another point B. The distance between B and the origin is 10. It follows that the coordinates of B could be either

(a) $\left(-5\sqrt{2}, 5\sqrt{2}\right)$ or $\left(5\sqrt{2}, -5\sqrt{2}\right)$,

(b) $(-4, 3)$ or $(4, -3)$,

(c) $\left(-5, 5\sqrt{3}\right)$ or $\left(5, -5\sqrt{3}\right)$,

(d) $(-8, 6)$ or $(8, -6)$,

(e) $\left(-5\sqrt{3}, 5\right)$ or $\left(5\sqrt{3}, -5\right)$.

统计

2018TMUA-paper1-17

17 There are two sets of data: the mean of the first set is 15, and the mean of the second set is 20.

One of the pieces of data from the first set is exchanged with one of the pieces of data from the second set.

As a result, the mean of the first set of data increases from 15 to 16, and the mean of the second set of data decreases from 20 to 17.

What is the mean of the set made by combining all the data?

A $16\frac{1}{4}$

B $16\frac{1}{3}$

C $16\frac{1}{2}$

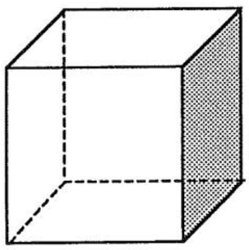
D $16\frac{2}{3}$

E $16\frac{3}{4}$

2000MAT-B

B. The faces of a cube are coloured red or blue. Exactly three are red and three are blue. The number of distinguishable cubes that can be produced (allowing the cube to be turned around) is

(a) 2 (b) 4 (c) 6 (d) 20.



2000MAT-J

J. A pack of cards consists of 52 different cards. A malicious dealer changes one of the cards for a second copy of another card in the pack and he then deals the cards to four players, giving thirteen to each. The probability that one players has two identical cards is

(a) $\frac{3}{13}$ (b) $\frac{12}{51}$ (c) $\frac{1}{4}$ (d) $\frac{13}{51}$.

2003MAT-I

I. You go into a supermarket to buy two packets of biscuits, which may or may not be of the same variety. The supermarket has 20 different varieties of biscuits and at least two packets of each variety. In how many ways can you choose your two packets?

(a) 400 (b) 210 (c) 200 (d) 190

逻辑题

2016TMUA-paper2-8

8 A region is defined by the inequalities $x + y > 6$ and $x - y > -4$

Consider the three statements:

1 $x > 1$

2 $y > 5$

3 $(x + y)(x - y) > -24$

Which of the above statements is/are true for **every** point in the region?

A none

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

2018TMUA-paper2-20

20 It is given that the equation $\sqrt{x+p} + \sqrt{x} = p$ has at least one real solution for x, where p is a real constant.

What is the complete set of possible values for p?

A $p = 0$ or $p = 1$

B $p = 0$ or $p \geq 1$

C $p \geq -x$

D $p \geq \sqrt{x}$

E $p \geq 0$

F $p \geq 1$

2016TMUA-paper2-9

9 Triangles ABC and XYZ have the **same area**.

Which of these extra conditions, taken independently, would **imply** that they are congruent?

(1) $AB = XY$ **and** $BC = YZ$

(2) $AB = XY$ **and** $\angle ABC = \angle XYZ$

(3) $\angle ABC = \angle XYZ$ **and** $\angle BCA = \angle YZX$

	Condition (1)	Condition (2)	Condition (3)
A	Does not imply congruent	Does not imply congruent	Does not imply congruent
B	Does not imply congruent	Does not imply congruent	Implies congruent
C	Does not imply congruent	Implies congruent	Does not imply congruent
D	Does not imply congruent	Implies congruent	Implies congruent
E	Implies congruent	Does not imply congruent	Does not imply congruent
F	Implies congruent	Does not imply congruent	Implies congruent
G	Implies congruent	Implies congruent	Does not imply congruent
H	Implies congruent	Implies congruent	Implies congruent

2017TMUA-paper2-13

13 The positive real numbers $a \times 10^{-3}$, $b \times 10^{-2}$ and $c \times 10^{-1}$ are each in standard form, and

$$(a \times 10^{-3}) + (b \times 10^{-2}) = (c \times 10^{-1}).$$

Which of the following statements (I, II, III, IV) **must** be true?

I $a > 9$

II $b > 9$

III $a < c$

IV $b < c$

A I only

B II only

C I and II only

D I and III only

E I and IV only

F II and III only

G II and IV only

H I, II, III and IV

2017TMUA-paper2-17

17 A set S of whole numbers is called *stapled* **if and only if** for every whole number a which is in S there exists a prime factor of a which divides at least one other number in S.

Let T be a set of whole numbers. Which of the following is true **if and only if** T is **not** stapled?

A For every number a which is in T, there is no prime factor of a which divides every other number in T.

B For every number a which is in T, there is no prime factor of a which divides at least one other number in T.

C For every number a which is in T, there is a prime factor of a which does not divide any other number in T.

D For every number a which is in T, there is a prime factor of a which does not divide at least one other number in T.

E There exists a number a which is in T such that there is no prime factor of a which divides every other number in T.

F There exists a number a which is in T such that there is no prime factor of a which divides at least one other number in T.

G There exists a number a which is in T such that there is a prime factor of a which does not divide any other number in T.

H There exists a number a which is in T such that there is a prime factor of a which does not divide at least one other number in T.

2017TMUA-paper2-20

20 I have forgotten my 5-character computer password, but I know that it consists of the letters `a`, `b`, `c`, `d`, `e` in some order. When I enter a potential password into the computer, it tells me exactly how many of the letters are in the correct position.

When I enter `abcde`, it tells me that none of the letters are in the correct position. The same happens when I enter `cdbea` and `eadbc`.

Using the best strategy, how many **further** attempts must I make in order to **guarantee** that I can **deduce** the correct password?

A None: I can deduce it immediately

B One

C Two

D Three

E More than three

2018TMUA-paper2-12

12 Consider the following statement:

> For any positive integer N there is a positive integer K such that $N(Km+1)-1$ is not prime for any positive integer m.

Which one of the following is the negation of this statement?

A For any positive integer N there is a positive integer K such that there is a positive integer m for which $N(Km+1)-1$ is prime.

B For any positive integer N there is a positive integer K such that there is a positive integer m for which $N(Km+1)-1$ is not prime.

C For any positive integer N there is a positive integer K such that for any positive integer m, $N(Km+1)-1$ is not prime.

D For any positive integer N, any positive integer K and any positive integer m, $N(Km+1)-1$ is not prime.

E There is a positive integer N such that for any positive integer K there is a positive integer m for which $N(Km+1)-1$ is not prime.

F There is a positive integer N such that for any positive integer K there is a positive integer m for which $N(Km+1)-1$ is prime.

G There is a positive integer N such that for any positive integer K and any positive integer m, $N(Km+1)-1$ is prime.

H There is a positive integer N and a positive integer K for which there is no positive integer m for which $N(Km+1)-1$ is prime.

2019TMUA-paper2-17

17 A multiple-choice test question offered the following four options relating to a certain statement:

A The statement is true **if and only if** $x > 1$

B The statement is true **if** $x > 1$

C The statement is true **if and only if** $x > 2$

D The statement is true **if** $x > 2$

Given that **exactly one** of these options was correct, which one was it?

2017TMUA-paper2-18

18 Consider the following problem:

Solve the inequality $\left(\frac{1}{4}\right)^n < \left(\frac{1}{32}\right)^{10}$, where n is a positive integer.

A student produces the following argument:

$$\left(\frac{1}{4}\right)^n < \left(\frac{1}{32}\right)^{10}$$

(I)

$$\log_{\frac{1}{2}}\left(\frac{1}{4}\right)^n < \log_{\frac{1}{2}}\left(\frac{1}{32}\right)^{10}$$

(II)

$$n\log_{\frac{1}{2}}\left(\frac{1}{4}\right) < 10\log_{\frac{1}{2}}\left(\frac{1}{32}\right)$$

(III)

$$n < \frac{10\log_{\frac{1}{2}}(\frac{1}{32})}{\log_{\frac{1}{2}}(\frac{1}{4})}$$

(IV)

$$n < \frac{10 \times 5}{2} = 25$$

(V)

$$1 \leqslant n \leqslant 24$$

Which step (if any) in the argument is invalid?

A There are no invalid steps; the argument is correct

B Only step (I) is invalid; the rest are correct

C Only step (II) is invalid; the rest are correct

D Only step (III) is invalid; the rest are correct

E Only step (IV) is invalid; the rest are correct

F Only step (V) is invalid; the rest are correct

2022TMUA-paper2-13

13 Consider the statement (∗) about a real number x:

(∗) **There exists** a real number y such that $x - xy + y$ is negative.

For how many real values of x is (∗) true?

A no values of x

B exactly one value of x

C exactly two values of x

D all except exactly two values of x

E all except exactly one value of x

F all values of x

2022TMUA-paper2-16

16 In this question, $a_1, \ldots, a_{100}$ and $b_1, \ldots, b_{100}$ and $c_1, \ldots, c_{100}$ are three sequences of integers such that

$$a_n \leq b_n + c_n$$

for each n.

Which of the following statements **must** be true?

I (minimum of $a_1, \ldots, a_{100}$) $\leq$ (minimum of $b_1, \ldots, b_{100}$) + (minimum of $c_1, \ldots, c_{100}$)

II (minimum of $a_1, \ldots, a_{100}$) $\geq$ (minimum of $b_1, \ldots, b_{100}$) + (minimum of $c_1, \ldots, c_{100}$)

III (maximum of $a_1, \ldots, a_{100}$) $\leq$ (maximum of $b_1, \ldots, b_{100}$) + (maximum of $c_1, \ldots, c_{100}$)

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

2020TMUA-paper2-3

3 A student makes the following claim:

For all integers n, the expression $4\left(\frac{9n+1}{2}-\frac{3n-1}{2}\right)$ is divisible by 3.

Here is the student's argument:

$$\begin{aligned}
4\left(\frac{9n+1}{2}-\frac{3n-1}{2}\right) &= 2\left(2\left(\frac{9n+1}{2}-\frac{3n-1}{2}\right)\right) && \text{(I)}\\
&= 2(9n+1-3n-1) && \text{(II)}\\
&= 2(6n) && \text{(III)}\\
&= 12n && \text{(IV)}\\
&= 3(4n) && \text{(V)}
\end{aligned}$$

which is always a multiple of 3. (VI)

So the expression $4\left(\frac{9n+1}{2}-\frac{3n-1}{2}\right)$ is always divisible by 3.

Which one of the following is true?

A The argument is correct.

B The argument is incorrect, and the first error occurs on line (I).

C The argument is incorrect, and the first error occurs on line (II).

D The argument is incorrect, and the first error occurs on line (III).

E The argument is incorrect, and the first error occurs on line (IV).

F The argument is incorrect, and the first error occurs on line (V).

G The argument is incorrect, and the first error occurs on line (VI).

2020TMUA-paper2-20

20 x is a real number and f is a function.

Given that **exactly one** of the following statements is true, which one is it?

A $x \geq 0$ **only if** $\text{f}(x) < 0$

B $x < 0$ **if** $\text{f}(x) \geq 0$

C $x \geq 0$ **only if** $\text{f}(x) \geq 0$

D $\text{f}(x) < 0$ **if** $x < 0$

E $\text{f}(x) \geq 0$ **only if** $x \geq 0$

F $\text{f}(x) \geq 0$ **if and only if** $x < 0$

2020TMUA-paper2-6

6 The function $\mathrm{f}(x)$ is defined for all real values of x.

Which of the following conditions on $\mathrm{f}(x)$ is/are **necessary** to ensure that

$$\int_{-5}^{0} \mathrm{f}(x)\,\mathrm{d}x = \int_{0}^{5} \mathrm{f}(x)\,\mathrm{d}x$$

Condition I: $\mathrm{f}(x) = \mathrm{f}(-x)$ for $-5 \le x \le 5$

Condition II: $\mathrm{f}(x) = c$ for $-5 \le x \le 5$, where c is a constant

Condition III: $\mathrm{f}(x) = -\mathrm{f}(-x)$ for $-5 \le x \le 5$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

2020TMUA-paper2-9

9 A student wishes to evaluate the function $\mathrm{f}(x) = x \sin x$, where x is in radians, but has a calculator that only works in degrees.

What could the student type into their calculator to correctly evaluate $\mathrm{f}(4)$?

A $(\pi \times 4 \div 180) \times \sin(4)$

B $(\pi \times 4 \div 180) \times \sin(\pi \times 4 \div 180)$

C $4 \times \sin(\pi \times 4 \div 180)$

D $(180 \times 4 \div \pi) \times \sin(4)$

E $(180 \times 4 \div \pi) \times \sin(180 \times 4 \div \pi)$

F $4 \times \sin(180 \times 4 \div \pi)$

2020TMUA-paper2-12

12 Which one of **A**–**F** correctly completes the following statement?

> Given that $a < b$, and $f(x) > 0$ for all x with $a < x < b$, the trapezium rule produces an overestimate for $\int_a^b f(x)\,dx$ ...

A ... **if** $f'(x) > 0$ **and** $f''(x) < 0$ for all x with $a < x < b$

B ... **only if** $f'(x) > 0$ **and** $f''(x) < 0$ for all x with $a < x < b$

C ... **if and only if** $f'(x) > 0$ **and** $f''(x) < 0$ for all x with $a < x < b$

D ... **if** $f'(x) < 0$ **and** $f''(x) > 0$ for all x with $a < x < b$

E ... **only if** $f'(x) < 0$ **and** $f''(x) > 0$ for all x with $a < x < b$

F ... **if and only if** $f'(x) < 0$ **and** $f''(x) > 0$ for all x with $a < x < b$

2020TMUA-paper2-14

14 An arithmetic sequence T has first term a and common difference d, where a and d are non-zero integers.

Property P is:

> **For some** positive integer m, the sum of the first m terms of the sequence is equal to the sum of the first $2m$ terms of the sequence.

For example, when $a = 11$ and $d = -2$, the sequence T has property P, because

$$11 + 9 + 7 + 5 = 11 + 9 + 7 + 5 + 3 + 1 + (-1) + (-3)$$

i.e. the sum of the first 4 terms equals the sum of the first 8 terms.

Which of the following statements is/are **true**?

I For T to have property P, it is **sufficient** that $ad < 0$.

II For T to have property P, it is **necessary** that d is even.

A neither of them

B I only

C II only

D I and II

2003MAT-J

J. There are real numbers x, y such that precisely *one* of the statements (a), (b),(c), (d) is true. Which is the true statement?

(a) $x \geq 0$
(b) $x < y$
(c) $x^2 > y^2$
(d) $|x| \leq |y|$

2021TMUA-paper2-11

11 A student attempts to solve the following problem, where a and b are non-zero real numbers:

Show that **if** $a^2 - 4b^3 \geq 0$ **then** there exist real numbers x and y such that $a = xy(x+y)$ and $b = xy$.

Consider the following attempt:

$(x-y)^2 \geq 0$ (I)

so $x^2 + y^2 - 2xy \geq 0$ (II)

so $(x+y)^2 - 4xy \geq 0$ (III)

so $x^2y^2(x+y)^2 - 4x^3y^3 \geq 0$ (IV)

so $a^2 - 4b^3 \geq 0$ (V)

Which of the following best describes this attempt?

A It is completely correct.

B It is incorrect, but it would be correct if written in the reverse order.

C It is incorrect, but the student has correctly proved the converse.

D It is incorrect because there is an error in line (II).

E It is incorrect because there is an error in line (III).

F It is incorrect because there is an error in line (IV).

2021TMUA-paper2-12

12 Which of the following statements about polynomials f and g is/are true?

I **If** $f(x) \geq g(x)$ for all $x \geq 0$, **then** $\int_0^x f(t)\,dt \geq \int_0^x g(t)\,dt$ for all $x \geq 0$.

II **If** $f(x) \geq g(x)$ for all $x \geq 0$, **then** $f'(x) \geq g'(x)$ for all $x \geq 0$.

III **If** $f'(x) \geq g'(x)$ for all $x \geq 0$, **then** $f(x) \geq g(x)$ for all $x \geq 0$.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

2022MAT-EXTRA-H

H. Suppose that a function $f(n)$ on the positive integers is defined such that $f(1) = 1$ and then for $n \geq 1$

$$f(2n) = f(n) \quad \text{and} \quad f(2n+1) = f(n) + f(n+1).$$

How many values of n are there such that $f(n) = 3$ and also n is a multiple of 35?

(a) 0,

(b) 1,

(c) 2,

(d) 3,

(e) Infinitely many.

2023MAT-EXTRA-G

G. For a pair of integers x and y with $x \geq 0$ and $y > 0$, we define

$$f(x, y) = \frac{1}{2}(x+y)(x+y+1) + y$$

What is the set of possible values that $f(x, y)$ can take?

(a) All positive integers.

(b) All positive even integers.

(c) All positive integers except for odd prime numbers.

(d) All positive integers that are triangular numbers (those which are the sum of the first k positive integers for some $k \geq 1$).

(e) All positive integers except for the triangular numbers.

SPECIMEN_TMUA_paper2-3

3. Consider the following attempt to solve an equation. The steps have been numbered for reference.

$$\sqrt{x+5} = x+3$$

(1)

$$x+5 = x^2+6x+9$$

(2)

$$x^2+5x+4=0$$

(3)

$$(x+4)(x+1)=0$$

$$x=-4 \text{ or } x=-1$$

Which one of the following statements is true?

A Both -4 and -1 are solutions of the equation.

B Neither -4 nor -1 are solutions of the equation.

C One solution is correct and the incorrect solution arises as a result of step (1).

D One solution is correct and the incorrect solution arises as a result of step (2).

E One solution is correct and the incorrect solution arises as a result of step (3).

SPECIMEN_TMUA_paper2-9

9. Consider the statement about Fred:

(*) *Every day next week, Fred will do at least one maths problem.*

If statement (*) is **not** true, which of the following is **certainly** true?

A Every day next week, Fred will do more than one maths problem.

B Some day next week, Fred will do more than one maths problem.

C On no day next week will Fred do more than one maths problem.

D Every day next week, Fred will do no maths problems.

E Some day next week, Fred will do no maths problems.

F On no day next week will Fred do no maths problems.

SPECIMEN_TMUA_paper2-17

17. Let S be a set of positive integers, for example S could consist of 3, 4, and 8.

A positive integer n is called an S-number **if and only if** for every factor m of n with $m > 1$, the number m is a multiple of some number in S.

So in the above example, 9 is an S-number; this is because the factors of 9 greater than 1 are 3 and 9, and each of these is a multiple of 3.

Positive integer n is therefore **not** an S-number **if and only if**

A for **every** (positive) factor m of n with $m > 1$, there is a number in S which is not a factor of m.

B for **every** (positive) factor m of n with $m > 1$, there is no number in S which is a factor of m.

C for **every** (positive) factor m of n with $m > 1$, every number in S is a factor of m.

D for **some** (positive) factor m of n with $m > 1$, there is a number in S which is not a factor of m.

E for **some** (positive) factor m of n with $m > 1$, there is no number in S which is a factor of m.

F for **some** (positive) factor m of n with $m > 1$, every number in S is a factor of m.

SPECIMEN_TMUA_paper2-18

18. A group of five numbers are such that:

- their mean is 0
- their range is 20

What is the largest possible median of the five numbers?

A 0

B 4

C $4\frac{1}{2}$

D $6\frac{1}{2}$

E 8

F 20

TMUA & MAT 好题集(until 2022)答案

微积分

2016TMUA-paper1-05: C
2018TMUA-paper1-12: F
2010MAT-I: B
2011MAT-G: D
2013MAT-C: C
2013MAT-E: D
2014MAT-J: A
1996MAT-H: (iii)
2000MAT-G: B
2021MAT-EXTRA-I: A
2020TMUA-paper1-11: E
2022MAT-EXTRA-G: C
2021TMUA-paper1-7: G
2021TMUA-paper1-15: C

画图

2017TMUA-paper1-18: A
2017TMUA-paper2-10: D
2018TMUA-paper2-15: G
2009MAT-F: D
2010MAT-J: D
2019TMUA-paper2-20: B
2015MAT-C: C
2022TMUA-paper1-18: B
2020TMUA-paper1-10: A
2018MAT-J: C
2022MAT-J: E
2021TMUA-paper2-16: E
2021TMUA-paper1-17: A
2021TMUA-paper1-18: B
2021TMUA-paper1-20: D
2020MAT-EXTRA-I: D
2020MAT-EXTRA-J: D
2021MAT-EXTRA-J: A
2023MAT-EXTRA-I: B
2023MAT-EXTRA-J: D
SPECIMEN_TMUA_paper1-10: D
SPECIMEN_TMUA_paper2-1: B

多项式

2018TMUA-paper1-8: D
2018TMUA-paper2-19: C
2008MAT-C: A
2019TMUA-paper1-14: B
2019TMUA-paper1-15: A
2009_SPECIMEN_MAT_2-H: B
2009_SPECIMEN_MAT_2-I: B
2008MAT-J: B
2009MAT-C: B
2009MAT-I: B
2010MAT-C: D
2010MAT-H: C
2011MAT-F: B
2011MAT-I:B
2012MAT-G: C

2013MAT-F: A
2013MAT-G: D
2014MAT-E: B
2016MAT-I: C
2017MAT-I: D
2015MAT-B: B
1997MAT-K: (iv)
1998MAT-D: (iv)
2000MAT-E: D
2020TMUA-paper1-8: D
2020TMUA-paper1-9: C
2020TMUA-paper1-17: A
2020TMUA-paper1-19: E
2020TMUA-paper2-18: G
2002MAT-A: A
2003MAT-A: C
2018MAT-G: B
2019MAT-F: C
2019MAT-G: A
2020MAT-I: B
2004MAT-A: B
2021MAT-EXTRA-E: E
2021TMUA-paper2-14: C
2021TMUA-paper2-17: F
2021TMUA-paper2-19: F
2022MAT-EXTRA-C: B
2023MAT-EXTRA-H: B
SPECIMEN_TMUA_paper2-14: B
SPECIMEN_TMUA_paper2-19: B

几何

2018TMUA-paper1-19: D
2019TMUA-paper1-18: B
2012MAT-J: B
2022TMUA-paper1-15: H
2020TMUA-paper1-16: C
2018MAT-H: C
2021TMUA-paper2-18: C
2021MAT-EXTRA-F: D

统计

2018TMUA-paper1-17: A
2000MAT-B: A
2000MAT-J: B
2003MAT-I: B

逻辑题

2016TMUA-paper2-8: B
2018TMUA-paper2-20: B
2016TMUA-paper2-9: D
2017TMUA-paper2-13: B
2017TMUA-paper2-17: F
2017TMUA-paper2-20: B
2018TMUA-paper2-12: F
2019TMUA-paper2-17: D
2017TMUA-paper2-18: B
2022TMUA-paper2-13: E
2022TMUA-paper2-16: D
2020TMUA-paper2-3: C

2020TMUA-paper2-20: C
2020TMUA-paper2-6: A
2020TMUA-paper2-9: F
2020TMUA-paper2-12: D
2020TMUA-paper2-14: A
2003MAT-J: D
2021TMUA-paper2-11: C
2021TMUA-paper2-12: B
2022MAT-EXTRA-H: A
2023MAT-EXTRA-G: E
SPECIMEN_TMUA_paper2-3: C
SPECIMEN_TMUA_paper2-9: E
SPECIMEN_TMUA_paper2-17: E
SPECIMEN_TMUA_paper2-18: E

TMUA2023-paper1-10

10 The trapezium rule with 4 strips is used to estimate the integral:

$$\int_{-2}^{2} \sqrt{4-x^2}\, dx$$

What is the positive difference between the estimate and the exact value of the integral?

A $2(\pi - 2 - 2\sqrt{3})$

B $2(\pi - 1 - \sqrt{3})$

C $2(2\pi - 1 - \sqrt{3})$

D $4(\pi - 1 - \sqrt{3})$

E $2\pi - 3\sqrt{3}$

F $4\pi - 6\sqrt{3}$

TMUA2023-paper1-11

11 It is given that $f(x) = x^2 - 6x$

The curves $y = f(kx)$ and $y = f(x - c)$ have the same minimum point, where $k > 0$ and $c > 0$

Which of the following is a correct expression for k in terms of c?

A $k = \frac{3-c}{3}$

B $k = \frac{3}{c+3}$

C $k = \frac{c-6}{6}$

D $k = \frac{6}{6-c}$

E $k = \frac{c+9}{9}$

F $k = \frac{9}{c-9}$

TMUA2023-paper1-12

12 How many solutions are there to the equation

$$\frac{2^{\tan^2 x}}{4^{\sin^2 x}} = 1$$

in the range $0 \le x \le 2\pi$?

A 2

B 3

C 4

D 5

E 6

F 7

G 8

TMUA2023-paper1-14

14 The function

$$f(x) = \frac{2}{3}x^3 + 2mx^2 + n, \quad m > 0$$

has three distinct real roots.

What is the complete range of possible values of n, in terms of m?

A $-\frac{8}{3}m^3 < n < 0$

B $-\frac{4}{3}m^3 < n < 0$

C $0 < n < \frac{3}{2}m^2$

D $0 < n < \frac{40}{3}m^3$

E $n < -\frac{8}{3}m^3$

F $n < \frac{3}{2}m^2$

G $n > -\frac{4}{3}m^3$

H $n > \frac{40}{3}m^3$

TMUA2023-paper1-15

15 The difference between the maximum and minimum values of the function $f(x) = a^{\cos x}$, where $a > 0$ and x is real, is 3.

Find the sum of the possible values of a.

A 0

B $\frac{3}{2}$

C $\frac{5}{2}$

D 3

E $\sqrt{10}$

F $\sqrt{13}$

TMUA2023-paper1-16

16 A right-angled triangle has vertices at (2, 3), (9, −1) and (5, k).

Find the sum of all the possible values of k.

A −8

B −6

C 0.25

D 2

E 2.25

F 8.25

G 10.25

TMUA2023-paper2-7

7 The graph of the line $ax + by = c$ is drawn, where a, b and c are real non-zero constants.

Which one of the following is a **necessary** but **not** **sufficient** condition for the line to have a positive gradient **and** a positive y-intercept?

A $\frac{c}{b} > 0$ and $\frac{a}{b} < 0$

B $\frac{c}{b} < 0$ and $\frac{a}{b} > 0$

C $a > b > c$

D $a < b < c$

E a and c have opposite signs

F a and c have the same sign

TMUA2023-paper1-17

17 A circle C_n is defined by

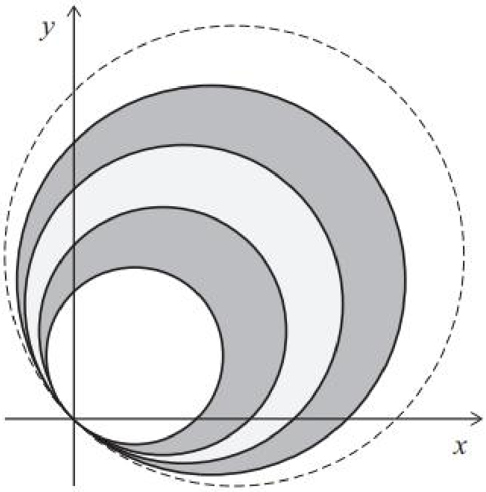
$$x^2 + y^2 = 2n(x + y)$$

where n is a positive integer.

C_1 and C_2 are drawn and the area between them is shaded.

Next, C_3 and C_4 are drawn and the area between them is shaded.

This is shown in the diagram.



[diagram not to scale]

This process continues until 100 circles have been drawn.

What is the total shaded area?

A 100π

B 500π

C 2500π

D 5050π

E 10100π

F 40400π

TMUA2023-paper1-18

18 You are given that

$$S = 4 + \frac{8k}{7} + \frac{16k^2}{49} + \frac{32k^3}{343} + \cdots + 4\left(\frac{2k}{7}\right)^n + \cdots$$

The value for k is chosen as an integer in the range $-5 \le k \le 5$

All possible values for k are equally likely to be chosen.

What is the probability that the value of S is a finite number greater than 3?

A $\frac{1}{11}$

B $\frac{1}{10}$

C $\frac{3}{11}$

D $\frac{3}{10}$

E $\frac{5}{11}$

F $\frac{1}{2}$

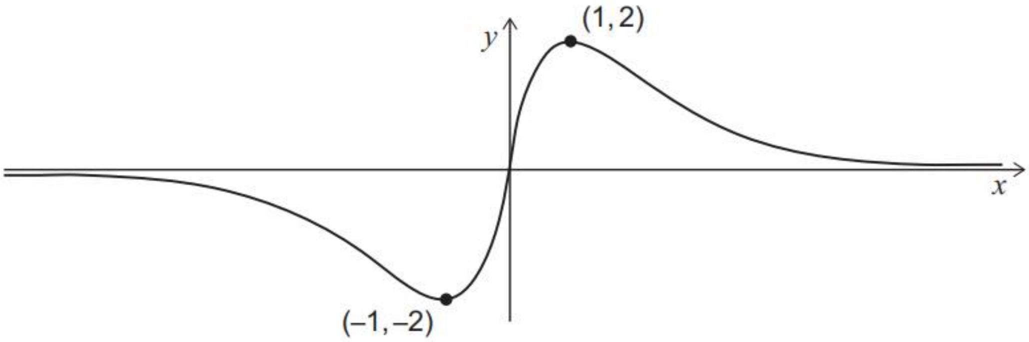
G $\frac{7}{11}$

H $\frac{7}{10}$

TMUA2023-paper1-20

20 The diagram shows the graph of $y = f(x)$

The function f attains its maximum value of 2 at $x = 1$, and its minimum value of -2 at $x = -1$



Find the difference between the maximum and minimum values of $(f(x))^2 - f(x)$

A 2

B $\frac{9}{4}$

C 4

D $\frac{17}{4}$

E 6

F $\frac{25}{4}$

G 8

H $\frac{33}{4}$

TMUA2023-paper2-18

18 The equation $x^4 + bx^2 + c = 0$ has four distinct real roots **if and only if** which of the following conditions is satisfied?

A $b^2 > 4c$

B $b^2 < 4c$

C $c > 0$ **and** $b > 2\sqrt{c}$

D $c > 0$ **and** $b < -2\sqrt{c}$

E $c < 0$ **and** $b < 0$

F $c < 0$ **and** $b > 0$

TMUA2023-paper2-4

4 A student attempts to answer the following question.

What is the largest number of consecutive odd integers that are all prime?

The student's attempt is as follows:

I There are two consecutive odd integers that are prime (for example: 17, 19).

II Any three consecutive odd integers can be written in the form $n-2$, n, $n+2$ for some n.

III If n is one more than a multiple of 3, then $n+2$ is a multiple of 3.

IV If n is two more than a multiple of 3, then $n-2$ is a multiple of 3.

V The only other possibility is that n is a multiple of 3.

VI In each case, one of the integers is a multiple of 3, so not prime.

VII Therefore the largest number of consecutive odd integers that are all prime is two.

Which of the following best describes this attempt?

A It is completely correct.

B It is incorrect, and the first error is on line I.

C It is incorrect, and the first error is on line II.

D It is incorrect, and the first error is on line III.

E It is incorrect, and the first error is on line IV.

F It is incorrect, and the first error is on line V.

G It is incorrect, and the first error is on line VI.

H It is incorrect, and the first error is on line VII.

TMUA2023-paper2-9

9 Consider the following statement about a pentagon P:

(∗) **If** at least one of the interior angles in P is 108°, **then** all the interior angles in P form an arithmetic sequence.

Which of the following is/are true?

I The statement (∗)

II The contrapositive of (∗)

III The converse of (∗)

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

TMUA2023-paper2-12

12 In this question, p is a real constant.

The equation $\sin x \cos^2 x = p^2 \sin x$ has n distinct solutions in the range $0 \le x \le 2\pi$

Which of the following statements is/are true?

I $n = 3$ is **sufficient** for $p > 1$

II $n = 7$ **only if** $-1 < p < 1$

A none of them

B I only

C II only

D I and II

TMUA2023-paper2-11

11 In this question, k is a positive integer.

Consider the following theorem:

If 2^k+1 is a prime, **then** k is a power of 2. $(*)$

Which of the following statements, taken individually, is/are equivalent to $(*)$?

I **If** k is a power of 2, **then** 2^k+1 is prime.

II 2^k+1 is **not** prime **only if** k is **not** a power of 2.

III A **sufficient** condition for k to be a power of 2 is that 2^k+1 is prime.

	Statement I is equivalent to $(*)$	Statement II is equivalent to $(*)$	Statement III is equivalent to $(*)$
A	Yes	Yes	Yes
B	Yes	Yes	No
C	Yes	No	Yes
D	Yes	No	No
E	No	Yes	Yes
F	No	Yes	No
G	No	No	Yes
H	No	No	No

TMUA2023-paper2-13

13 Let x be a real number.

Which **one** of the following statements is a **sufficient** condition for **exactly** three of the other four statements?

A $x \geq 0$

B $x = 1$

C $x = 0$ **or** $x = 1$

D $x \geq 0$ **or** $x \leq 1$

E $x \geq 0$ **and** $x \leq 1$

TMUA2023-paper2-19

19 In this question, $f(x)$ is a non-constant polynomial, and $g(x) = xf'(x)$

$f(x) = 0$ for exactly M real values of x.

$g(x) = 0$ for exactly N real values of x.

Which of the following statements is/are true?

I It is possible that $M < N$

II It is possible that $M = N$

III It is possible that $M > N$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

TMUA2023-paper2-20

20 Let f be a polynomial with real coefficients.

The integral $I_{p,q}$ where $p < q$ is defined by

$$I_{p,q} = \int_p^q (f(x))^2 - (f(|x|))^2 \, dx$$

Which of the following statements must be true?

1 $I_{p,q} = 0$ **only if** $0 < p$

2 $f'(x) < 0$ for all x **only if** $I_{p,q} < 0$ for all $p < q < 0$

3 $I_{p,q} > 0$ **only if** $p < 0$

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

TMUA2023-paper1-10: B
TMUA2023-paper1-11: B
TMUA2023-paper1-12: F
TMUA2023-paper1-14: A
TMUA2023-paper1-15: F
TMUA2023-paper1-16: E
TMUA2023-paper2-7: E
TMUA2023-paper1-17: E
TMUA2023-paper1-18: E
TMUA2023-paper1-20: F
TMUA2023-paper2-18: D
TMUA2023-paper2-4: G
TMUA2023-paper2-9: D
TMUA2023-paper2-12: C
TMUA2023-paper2-11: G
TMUA2023-paper2-13: C
TMUA2023-paper2-19: H
TMUA2023-paper2-20: D

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THE ULTIMATE CAMBRIDGE TMUA COLLECTION 好题整理

如果你还没做这本书, 别做! 快跑! 是狗屎! 做完这些就行!

MOCK1 PAPER2

5. Let I, II, III, IV be some statements. Suppose that $I \to II \to III$ and $IV \to Not\ III$ and $Not\ I \to II$, where $a \to b$ means if a is true, then b is true. $not\ a$ is just the opposite to a, so if a is true, $not\ a$ is false and vice versa.

Suppose II is a true statement. What can we say about the rest of the statements?

	I	III	IV
A	true	true	true
B	could be either	true	could be either
C	could be either	true	false
D	false	true	false
E	true	false	false

8. Let I and II be two statements. You are asked to show that I if and only if II. Which of the following does not prove the statement?

A. II if I, and I if II

B. $not\ I$ if II, and $not\ II$ if I

C. $not\ II$ if $not\ I$, and $not\ I$ if $not\ II$

D. $not\ I$ if $not\ II$, and I if II

17. A student attempts to solve the following equation:

$$\frac{x^2-5x+6}{x^2+x+1} = \frac{x^2-5x+6}{2x^2-3x-2},$$

by using the following steps:

a) $\frac{1}{x^2+x+1} = \frac{1}{2x^2-3x-2}$

b) $x^2+x+1 = 2x^2-3x-2$

c) $x^2+2x-3=0$

d) $x=-3,1$

Which of the following best describes the solution?

A. The method is completely correct

B. The method is incorrect and from a to b we have introduced extra solutions

C. The method is incorrect and we are missing one solution

D. The solutions given are incorrect

E. The method is incorrect and we are missing two solutions

MOCK2 PAPER1

17. The graph of $y=\sqrt{5x-2}$ undergoes the below transformations in the given order.

I Translated horizontally left by 4

II Translated vertically down by 6

III Vertical reflection in the axis y = 0

IV Stretch factor $\frac{1}{3}$ in the y-axis

V Stretch factor 2 in the x-axis

Which of the following equations describes the transformed graph?

A. $y=\sqrt{2-\frac{5x}{18}}-2$

B. $y=\frac{\sqrt{-10x-6}-4}{3}$

C. $y=-\frac{\sqrt{\frac{5}{2}x-22}-6}{3}$

D. $y=-\frac{\sqrt{\frac{5}{2}x+18}-6}{3}$

E. $y=\frac{\sqrt{\frac{5}{2}x-22}-6}{3}$

F. $y=-\frac{\sqrt{\frac{5}{2}x+22}-6}{3}$

G. $y=\frac{\sqrt{\frac{5}{2}x+14}}{3}$

MOCK2 PAPER2

16. Consider the following attempt to solve the equation $x = \sqrt{2x+5}$

Line 1: $x = \sqrt{2x+5}$
Line 2: $x^2 = 2x+5$
Line 3: $x^2 - 2x - 5 = 0$
Line 4: $(x-1)^2 - 6 = 0$
Line 5: $(x-1)^2 = 6$
Line 6: $x - 1 = \pm\sqrt{6}$
Line 7: $x = 1 \pm \sqrt{6}$

Which of the following is true?

A. the solutions are both correct
B. only one of the solutions is correct due to an error in line 3
C. only one of the solutions is correct due to an error in line 4
D. neither of the solutions are correct due to an error in line 2
E. only one of the solutions is correct due to errors in lines 2 and 4
F. only one of the solutions is correct due to errors in lines 2 and 3
G. only one of the solutions is correct due to an error in line 2
H. neither of the solutions are correct due to an error in line 4

MOCK3 PAPER1

11. For a function $f(x)$, it is given that $\left(\int_0^2 f(x)dx\right)\left(\int_0^1 f(x)dx + 4\right)$=24. You are also told that, for this function, $f(x+1) = f(1-x)$.

Which of the following could the value of $\int_1^2 f(x)dx$ be?

I. -6
II. 8
III. 2

A. None
B. I only
C. II only
D. III only
E. I & II only
F. I & III only
G. II & III only
H. All three

13. A hospital has two wards, one for critically ill patients called Ward A and one for general patients, called Ward B. The probability a patient in Ward A survives is $\frac{1}{9}$, and the probability a patient in Ward B survives is $\frac{3}{5}$. (Every patient is in one of the two wards).

Given that there are p people in Ward A, and the overall survival rate of the hospital is $\frac{1}{4}$, what is q, the number of people in Ward B, in terms of p?

A. $\frac{3p}{45}$
B. $\frac{45p}{3}$
C. $\frac{25p}{63}$
D. $\frac{63p}{25}$
E. 50
F. $\frac{45p}{12p-45}$
G. $\frac{12p-45}{45p}$

15. I have 2 blue balls and 3 red balls which I will throw into 5 indistinguishable boxes. Assuming I don't miss, how many distinct situations could I be in after all the balls are thrown?

A. 14
B. 5^5
C. 32
D. 16
E. 8
F. 4
G. $5^3 + 5^2$

MOCK4 PAPER1

1. Consider the parabola $y = (x-3)^2 - 4$ and the line $y = mx + c$. Let R be the size of the interval m must lie in such that the line and parabola do not intersect. What is c in terms of R?

A. $c = 5 - R^2$

B. $c = 5 - \frac{(R+6)^2}{4}$

C. $c = 5 - \frac{R^2}{16}$

D. $c = 20 - \frac{R^2}{4}$

E. $c = 20 - (R+6)^2$

MOCK4 PAPER2

3. Consider the following proof by contradiction that $\sqrt{12}$ is irrational.

(I) Suppose $\sqrt{12}$ is rational. The we can express it in the form $\sqrt{12} = \frac{a}{b}$ for some a and b with no common divisors.

(II) $12 = \frac{a^2}{b^2}$

(III) $12b^2 = a^2$

(IV) 12 divides a^2, which means that 12 must also divide a i.e. $a = 12c$ for some integer c.

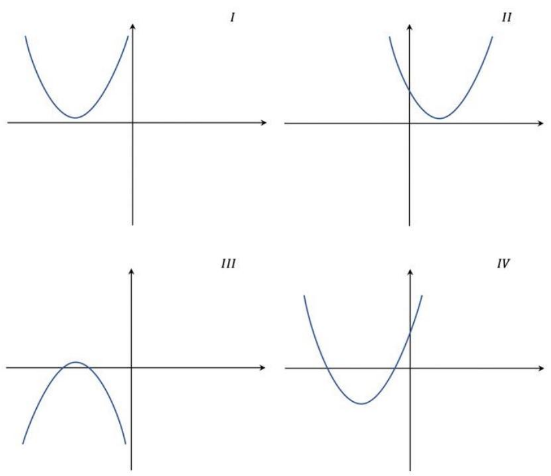
(V) This means $144c^2 = 12b^2$ i.e. $b^2 = 12c^2$

(VI) Using the logic of line (IV), $b = 12d$ for some integer d.

(VII) But then a and d both have 12 as a divisor, which is a contradiction.

A. All correct
B. Line (I)
C. Line (II)
D. Line (IV)
E. Line (V)
F. Line (VI)
G. Line (VII)

5. Consider the graph of the curve $y = a(x+b)^2 + c$ where all of $a, b, c > 0$. Now suppose that a decreases and c increases. Which of the following graphs can it not be now?



A. I only
B. II only
C. III only
D. IV only
E. I & II only
F. I & III only
G. II & IV only
H. III & IV only

8. Consider this student's attempt at finding the solutions to $\sqrt{x+3} = 3x - 1$:

(I) $x + 3 = (3x - 1)^2$
(II) $x + 3 = 9x^2 - 6x + 1$
(III) $0 = 9x^2 - 7x - 2$
(IV) $0 = (x - 1)(9x + 2)$
(V) So $x = 1 \, and - 2/9$

Is this correct?

A. Both answers are correct.
B. Only one is right and it's due to an error in line (I).
C. Only one is right and it's due to an error in lines (II) and (III).
D. Only one is right and it's due to an error in line (IV).
E. Neither is right and it's due to an error in line (I).
F. Neither is right and it's due to an error in lines (II) and (III).
G. Neither is right and it's due to an error in line (IV).

11. Let $f(x) = ax^2$ and let $\int_{-b}^{b} f(x)dx = R$. Find the area between the curve $f(x - b) - f(b)$ and the x-axis.

A. $f(b) - R$
B. $f(b) - 2R$
C. $3R + f(b) + b$
D. $f(b) - 2bR$
E. R
F. $2bf(b) - R$

13. Consider the following proof by induction that $3 \times 7^n + 6$ is divisible by 9 for all non-negative n:

(I) Check the base case, $n = 1$; $3 \times 7 + 6 = 27$, which is indeed divisible by 9.
(II) First, we suppose $3 \times 7^n + 6$ is divisible by 9 for $n \leq k$. i.e. $3 \times 7^k + 6 = 9M$ for some integer M.
(III) Then $3 \times 7^{k+1} + 6 = 7 \times (3 \times 7^k) + 6$
(IV) This means $7 \times (3 \times 7^k) + 6 = 7(9M - 6) + 6$
(V) Which means $3 \times 7^{k+1} + 6 = 9(7M - 4)$
(VI) So $3 \times 7^{k+1} + 6$ is also divisible by 9, which means $3 \times 7^n + 6$ is divisible by 9 for all non-negative n

Where is the first error?

A. The proof is correct.
B. Line (I)
C. Line (II)
D. Line (III)
E. Line (IV)
F. Line (V)
G. Line (VI)

17. I have a bag with many tiles in, each of which has one integer written on it. I pick a tile out of the bag at random. $P(\text{the number is odd}) = \frac{1}{3}$ and $P(\text{the number is prime}) = \frac{1}{5}$.

Consider the following statements:

I. $P(\text{number is even}) - P(\text{the number is } 2) = \frac{4}{5}$
II. $P(\text{the number is not prime and is even}) = \frac{8}{15}$
III. $P(\text{the number is not prime and is odd}) = \frac{2}{15} + P(\text{the number is } 2)$

Which of these statements are definitely true?

A. None
B. I only
C. II only
D. III only

18. Suppose that the equation $\sqrt{xp} = x + \sqrt{p}$ has exactly 1 solution for x. How many valid values of p are there?

A. Infinitely many (i.e. an interval)
B. 4
C. 3
D. 2
E. 1
F. 0
G. None

MOCK1 PAPER2

Q5: C
Q8: B

Q17: **C**

The error is in cancelling the numerator at the start. The numerator could be 0. We can amend this by instead beginning by cross multiplying: $(2x^2 - 3x - 2)(x^2 - 5x + 6) = (x^2 + x + 1)(x^2 - 5x + 6)$, and so $(x^2 - 5x + 6)(x^2 + 2x - 3) = 0$, $(x - 3)(x - 2)(x + 3)(x - 1) = 0$, giving four solutions.

实际上 **x=2** 不可能为根**(**分母要不然就是 **0** 了**)**, 因此选 **C**

MOCK2 PAPER1

Q17: **D**

- Translated horizontally left by 4
  - Replace x with $(x + 4)$
  - $y = \sqrt{5(x + 4) - 2}$
  - $y = \sqrt{5x + 18}$
- Translated vertically down by 6
  - Replace y with $y + 6$
  - $y + 6 = \sqrt{5x + 18}$
- Vertical reflection in the axis $y = 0$
  - Replace y with $-y$
  - $-y = \sqrt{5x + 18} - 6$
- Stretch factor $\frac{1}{3}$ in the y-axis
  - Replace y with $3y$
  - $-3y = \sqrt{5x + 18} - 6$
- Stretch factor 2 in the x-axis
  - $-3y = \sqrt{5(\frac{1}{2}x) + 18 - 6}$

Thus, $y = -\frac{\sqrt{\frac{5}{2}x + 18 - 6}}{3}$

MOCK2 PAPER1

Q16: G

MOCK3 PAPER1

Q11: **F**

The function is symmetric about 1, which means $\int_1^2 f(x)dx = \int_0^1 f(x)dx$. This implies that $\int_0^2 f(x)dx = 2\int_0^1 f(x)dx$. Making this substitution, we get a quadratic in $\int_0^1 f(x)dx$ with $\left(\int_0^1 f(x)dx\right)^2 + 4\int_0^1 f(x)dx - 12 = 0 = (\int_0^1 f(x)dx + 6)(\int_0^1 f(x)dx - 2)$. This gives us $\int_0^1 f(x)dx = \int_1^2 f(x)dx = 2$ or $\int_1^2 f(x)dx = -6$.

Q13: **C**

Possibly the simplest way to consider this problem is to look at the number of patients who will survive. From splitting the wards up, we know that the number of patients who will survive is $\frac{3q}{5} + \frac{p}{9}$. We also know it is $\frac{p+q}{4}$ by taking the number of people in the whole hospital, and the overall survival rate. By rearranging the fractions, we get $108q + 20p = 45p + 45q$; $63q = 25p; q = \frac{25p}{63}$. And q was the number we were looking for.

Q15: **D**

This is a problem about counting. The boxes are indistinguishable, which means we only care about how the balls are distributed, not specifically which boxes they are in.
A good way to approach these is to have an order to the counting. We shall do it based on how many boxes are empty at the end:

0 – This is only true if there is 1 ball in each box, and there is only one way to do this.

1 – There must be two balls in 1 box and 1 in the other 3. The two balls could both be blue, both be red, or be 1 of each.

2 – There are two possibilities here, a $3, 1, 1$ split, or a $2, 2, 1$ split. In the former case, the 3 balls could be all red, 2 red 1 blue, or all blue. Either way they uniquely determine the other balls. In the latter case, we consider where the two blue balls are; they could be in the same box, one could be in each of the boxes with 2 balls in, or one could be in a 2 box and the other in the 1 box. As there are no other ways to distribute the blue balls, this is an exhaustive list.

3 – All balls being only in 2 boxes means they are split $1, 4$ or $2, 3$. In the former, there are 2 possibilities, the single ball is red, or it is blue. In the latter, there are 3 possibilities for the box with two balls in, RR, BB or RB, but no matter what this also determines what is in the other box.

4 – Only one way, all the balls are in the same box

Summing these possibilities, we have 16 ways.

MOCK4 PAPER1

Q1: **C**

To find crossing points, we equate the two lines.

$mx + c = (x - 3)^2 - 4 = x^2 - 6x + 5$
$0 = x^2 - (m + 6)x + 5 - c$

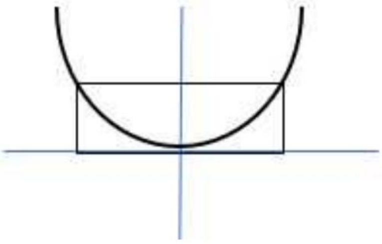
The lines do not cross so there are no real solutions, thus $b^2 - 4ac < 0$ i.e. $(m + 6)^2 - 20 + 4c = m^2 + 12m + 16 + 4c < 0$. We want the values of m which are at the edges of the interval which are $m = \frac{-12 \pm \sqrt{144-64-16c}}{2} = -6 \pm \sqrt{20 - 4c}$. Thus, the size of the interval is $R = 4\sqrt{5 - c}$. Rearranging we get $c = 5 - \frac{R^2}{16}$.

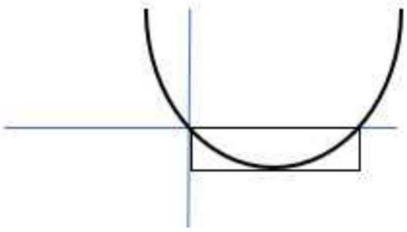
MOCK4 PAPER2

Q3: D
Q5: G
Q8: B

Q11: **F**

We know that $f(x)$ is shaped roughly like the graphs shown. The 2^{nd} graph is $f(x-b)-f(b)$, and the box in both graphs is the same size; it is $2b$ in length, and $f(b)$ in height. It is also clear to see that it encloses all of the area between the curve and the x-axis in the second picture. The excess being precisely what is described as R. Thus, the new area is $2bf(b)-R$.

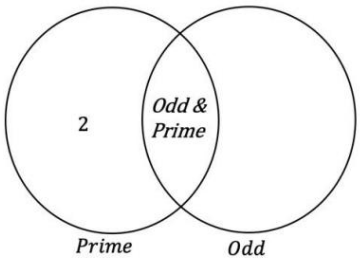




Q13: B (non-negative integers n)

Q17: **D**

A Venn diagram often helps in situations with probabilities which are not mutually exclusive.



I is the same as the entire outside of the Venn diagram, but because the properties of being prime and being odd are not mutually exclusive, we cannot multiply them to obtain the probability they occur at the same time. So, this is not true unless the only prime number in the bag is 2, and we don't know whether this is true or not.

II is in fact the same statement as I, which is clear by looking at the Venn diagram. So, once again, as we do not know anything about the probability of picking 2 compared to an odd prime, we cannot determine this; it is not necessarily true.

III is the probability that the number picked is odd and not prime, which means it inhabits the "odd" circle in the diagram, minus the intersection. We know that the probability of being in this intersection is $P(\text{prime})-P(2)=\frac{1}{5}-P(2)$. Thus, $P(\text{not prime and odd})=\frac{1}{3}-\left(\frac{1}{5}-P(2)\right)=\frac{2}{15}+P(2)$.

Thus, only III is always true.

Q18: **D**

As usual for surds questions, we can start by squaring both sides to obtain $px=x^2+2x\sqrt{p}+p;\ \ 0=x^2+(2\sqrt{p}-p)x+p$. This has exactly one solution so its discriminant must be 0 i.e. $(2\sqrt{p}-p)^2-4p=0=p^2-4p\sqrt{p}+4p-4p=p(p-4\sqrt{p})=p\sqrt{p}(\sqrt{p}-4)$. This tells us there are two solutions; $\sqrt{p}=0\ or\ 4; p=0\ or\ p=16$. 0 is obviously a valid solution, but 16 might not be, since we may have generated extra solutions. If we substitute in $p=16$, we must solve the equation $4\sqrt{x}=x+4; 0=x-4\sqrt{x}+4=(\sqrt{x}-2)^2=0$. This has precisely one solution, so $p=16$ is a valid solution as well.

We have 2 valid solutions for p.

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6.
During which of the following interval does

$$f(x) = \sin(\cos(x)) + 2\cos(x)$$

always increasing

(A) All real number x

(B) $(0, \frac{\pi}{2})$

(C) $(0, \pi)$

(D) $(\pi, 2\pi)$

(E) $(\frac{\pi}{2}, \frac{3\pi}{2})$

(F) $(\frac{\pi}{2}, \pi)$

8.
Given $\log_{10} 2 = 0.3010$, How many digits does 5^{30} has?
[Hints: consider log base 10]

(A) 9

(B) 91

(C) 20

(D) 21

(E) I totally have no idea, 投降扣一半

10.
For some real number x, y. It is given that

$$2^{x} - \frac{4}{2^{3y}} = 3^{-x} - \frac{3^{3y}}{9}$$

Which the following can we deduce

(A) $x > y$

(B) $x < y$

(C) $0 < xy \leq \frac{1}{3}$

(D) $x + 3y = 2$

11.
It is given there are only two distinct pairs of (x,y) simultaneously satisfy exactly **TWO** of following equations

$$x + y = 100$$
$$\sin(\theta)x + \cos(\theta)y = 3$$
$$-\cos(\theta)x + \sin(\theta)y = 2$$

What are the number of possible θ can ensure the statement holds between 0 to 2π.

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

(G) 6

12.
A circle is inscribed in the quadrilateral ABCD so that it touches all four sides

Sides AB and DC are parallel with lengths 2cm and 4cm,respectively

Sides AD and BC have equal length.

What,in centimetres,is the length of AC?

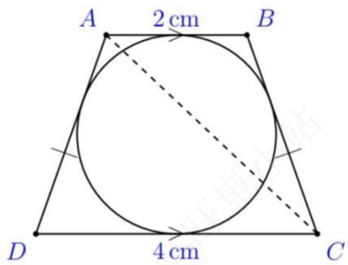
(A) √17

(B) 2√5

(C) √13

(D) 3√2

(E) 5



13.

Assume that u and v are the functions that satisfy:

u(0)=1, u(1)=2, u(2)=3,

v(0)=2, v(1)=3, v(2)=4,

u'(0)=4, u'(1)=5, u'(2)=6,

v'(0)=7, v'(1)=8, v'(2)=9.

Let f(x)=u(v(x)).Find $f'(0)$

(A) 21

(B) 7

(C) 14

(D) 16

(E) 24

(F) 42

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14.

Alex writes down the value of the following sum,

where the final term is the number consisting of 2024 consecutive nines:

$$9 + 99 + 999 + 9999 + \cdots + \underbrace{99\ldots9}_{\text{2023 nines}} + \underbrace{99\ldots9}_{\text{2024 nines}}$$

How many times does the digit 1 appear in the answer?

(A) 2019

(B) 2020

(C) 2022

(D) 2023

(E) 2024

(F) 2025

15.

We know the function $f(x) = \cos(x)$
When going through several transformation, will result in $g(x) = \cos(3(|x - 1|)$
Which of the following series of transformation is capable of getting f(x) to g(x)?

(A) Move f horizontally to the right by 1, and then reflect the positive part of the function to the left, and finally stretch the x axis by factor of $\frac{1}{3}$.

(B) Reflect the positive part of the function to the left, and then move f horizontally to the right by 1, and finally stretch the x axis by factor of $\frac{1}{3}$.

(C) Stretch the x axis by factor of $\frac{1}{3}$, and then reflect the positive part of the function to the left, and finally move f horizontally to the right by 1.

(D) Stretch the x axis by factor of $\frac{1}{3}$, and then reflect the positive part of the function to the left, and finally move f horizontally to the right by 1.

(E) Reflect the positive part of the function to the left, and then stretch the x axis by factor of 3, and finally move f horizontally to the right by 1.

16.

Believe it or not the following function is constant in an interval |a,b|.

$$f(x) = \sqrt{x + 2\sqrt{x-1}} + \sqrt{x - 2\sqrt{x-1}}$$

Find the interval where the function is constant.

A. [1,2]

B. [1,3]

C. [1,5]

D. [1,9]

E. [1,10]

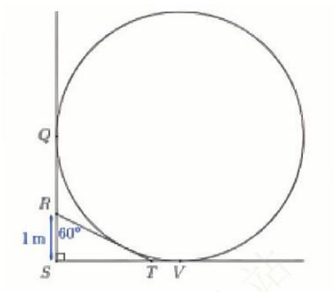
17.

In the diagram,QS,RT and SV are tangents to the circle.

The length of RS is 1 m

∠SRT=60°

What is the diameter of the circle,in metres?



(A) $3 + \sqrt{3}$

(B) $3\sqrt{3}$

(C) 4

(D) $2 + 2\sqrt{3}$

(E) $\frac{9}{2}$

18.

Find the remainder when $x^{203} - 1$ is divided by $x^4 - 1$

(A) $x^3 + 1$

(B) $x^3 - 1$

(C) $x^3 - x^2 + x - 1$

(D) $-x^3 + x^2 - x + 1$

(E) Mie...Mie...Mie

19.

We define f(n) as the following:

$$\mathrm{f(n)} = \int_{0}^{10} \lfloor nx \rfloor \, dx$$

Where n is an positive integer. We also define d(n) as the following

$$\mathrm{d(n)} = \int_{0}^{10} nx \, dx - \mathrm{f(n)}$$

Determine which of the following statement is true:

a. When n increases, f(n) increases.
b. When n increases, d(n) does not change
c. The series {f(n)-f(n-1)} forms an arithmetic series

(A) Only statement a is correct.

(B) Only statement b is correct.

(C) Only statement c is correct.

(D) Only statement a and b are correct.

(E) Only statement a and c are correct.

(F) Only statement b and c are correct.

(G) Statement a, b and c are all correct.

旺崽小站-TMUA MOCK PAPER2

2.

Five positive integers have a mean of 5,a median of 5 and just one mode of 8. What is the difference between the largest and the smallest integers in the set?

(A) 7

(B) 4

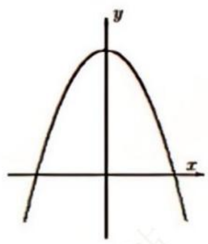
(C) 5

(D) 6

(E) 9

4.

The graph of $y=ax^2+bx+c$ is shown, with its vertex on the y-axis. Which of the following statements must be true?



(A) a+b-c<0

(B) -a+b-c>0

(C) a+b+c=0

(D) a+b+c<0

(E) there is not enough information

8.

There are four cards on the table, each has two sides with one letter on the front and one number on the back. It can be seen that the four sides showing on the desk are D, K, 3, and 7. Jack argues

"if one side of the card is 3, then the other side of the card is D".

Your task is to verify whether Jack' s statement is right or wrong, which card you should flip under optimal strategy?

(A) All four cards have to be flipped.

(B) Just flip the card with 3 on it was sufficient.

(C) Just flip the card with D on it was sufficient.

(D) Flip the two cards with 3 and D on it were sufficient.

(E) Flip the two cards with 3 and K on it were sufficient.

(F) Flip the three cards with 3, D and K on it were sufficient.

9.

$$f(x) = \log_2 (x + 2)$$
$$g(y) = (y - 8)^2 + c$$

We know for every x on [0, 6], there exists some y on [7, 10] such that g(y)>f(x). Find the range of c

(A) (-2,-1)

(B) (-1,3)

(C) (-3,∞)

(D) (-1,∞)

(E) (0,1)

(F) (0,∞)

12.
We define an planet to be **SMARTY** if and only if for every cool Aoteman on that planet, given any Guaishou that the Aoteman is battling with, we have the phenomenon that if Aoteman does not has a red button bling bling on his chest, then the Guaishou is not going to lose the battle.
Now, if we found a planet that is **NOT SMARTY**, which of the following argument is correct?

(A) for some cool Aoteman on that planet, there is some Guaishou that the Aoteman is battling with such that we have the phenomenon that Aoteman has a red button bling bling on his chest and the Guaishou is winning the battle

(B) for some cool Aoteman on that planet, there is some Guaishou that the Aoteman is battling with such that we have the phenomenon that Aoteman does not has a red button bling bling on his chest and the Guaishou is losing the battleweek

(C) for some cool Aoteman on that planet, given any Guaishou that the Aoteman is battling with, we have the phenomenon that Aoteman has a red button bling bling on his chest and the Guaishou is losing the battle

(D) for some cool Aoteman on that planet, given any Guaishou that the Aoteman is battling with, we have the phenomenon that Aoteman does not has a red button bling bling on his chest and the Guaishou is losing the battle

(E) for some cool Aoteman on that planet, given any Guaishou that the Aoteman is battling with, we have the phenomenon that if Aoteman does not has a red button bling bling on his chest, then the Guaishou is losing the battle

13.
Min{a,b} is taking the smaller values such that given two given values a and b. Now define

$$f(x) = \min\{|2x - 9| + c, x^2 - 2x + c\}$$

If f(x)=0 has four distinct roots, find the range of c

(A) (-3, 0)

(B) (-3, 1)

(C) (0, 3)

(D) (1, 3)

(E) (-1, 1)

(F) (-3, 3)

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(6) D

(8) D

10.

For some real number x, y. It is given that

$$2^x - \frac{4}{2^{3y}} = 3^{-x} - \frac{3^{3y}}{9}$$

Which the following can we deduce

(A) $x > y$

(B) $x < y$

(C) $0 < xy \leq \frac{1}{3}$

(D) $x + 3y = 2$

11.

It is given there are only two distinct pairs of (x,y) simultaneously satisfy exactly **TWO** of following equations

$$x + y = 100$$
$$\sin(\theta)x + \cos(\theta)y = 3$$
$$-\cos(\theta)x + \sin(\theta)y = 2$$

What are the number of possible θ can ensure the statement holds between 0 to 2π.

(A) 0

(B) 1

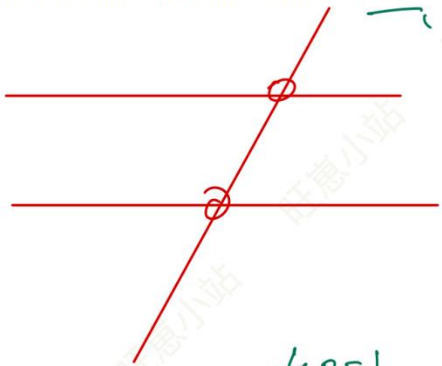
(C) 2

(D) 3

(E) 4

(F) 5

(G) 6



(12) A

(13) F

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14.

Alex writes down the value of the following sum,

where the final term is the number consisting of 2024 consecutive nines:

$$9 + 99 + 999 + 9999 + \cdots + \underbrace{99\ldots9}_{2023\text{ nines}} + \underbrace{99\ldots9}_{2024\text{ nines}}$$

How many times does the digit 1 appear in the answer?

(A) 2019

(B) 2020

(C) 2022

(D) 2023

(E) 2024

(F) 2025

(15) C or D

(16) A

(17) A

(18) B

(19) G

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(2) 答案是 8

(4) A

(8) B

(9) D

(12) B

(13) A

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R2Drew2 TMUA MOCK TEST A PAPER 1

4. What is the sum of all real solutions of the equation $2\sqrt{a} + \frac{7}{\sqrt{a}} = 9 - \frac{6}{a}$

A $4\frac{1}{2}$ B 5 C $12\frac{1}{2}$ D 13 E $13\frac{1}{4}$

17. There are two sets of data: the first set has 10 pieces of data and the second set has 20 pieces of data. The mean of the second set is four more than the mean of the first set.

One of the pieces of data from the first set is exchanged with a piece from the second set. As a result the mean of the second set is now one more than the mean of the first set.

Following this exchange, by how much has the mean of the first set increased?

A 0.5 B 1 C 2 D 4 E 10

20. The curve C has equation $y = x^3 - x^4$

The straight line L is a tangent to C at two distinct points. Find the equation of L.

A $y = \frac{1}{8}x + \frac{1}{64}$

B $y = \frac{1}{8}x - \frac{1}{64}$

C $y = \frac{1}{2}x + \frac{1}{8}$

D $y = \frac{1}{2}x - \frac{1}{8}$

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13. Consider the following two statements about the polynomial $p(x)$ with two stationary points at $x = a$ and $x = b$ where $a < b$

P: $p(x)$ has a maximum at $x = a$

Q: $p'(x) < 0$ for $a < x < b$

Which of the following is correct?

A P is **necessary** and **sufficient** for Q

B P is **not necessary** and **not sufficient** for Q

C P is **necessary** but **not sufficient** for Q

D P is **sufficient** but **not necessary** for Q

17. Which one of the following is a **necessary and sufficient** condition for the line with equation $y = mx + c$ to be a tangent to the circle with equation $x^2 + y^2 = a^2$

A $m = 0$ and $c = a$

B $m = 0$ and $c = \pm a$

C $m^2 + c^2 = a^2$

D $c^2 = \dfrac{a^2}{m^2}$

E $c^2 = a^2(m^2 + 1)$

20. Let x be a real number, and consider the inequality $x^2 + 1 \geq 10$

Which of the following conditions on x is **necessary but not sufficient** for this to be true?

A $x \leq -3$ or $x \geq 3$

B $x \neq 0$

C $x > 10$

D $x = 3$

E $x > 0$

R2Drew2 TMUA MOCK TEST B PAPER 1

2. How many solutions does the following equation have in the range $0 \leq x \leq 2\pi$

$$sin2x + sin^2x = 1$$

A 2

B 3

C 4

D 6

E 8

F infinitely many

5. Find the sum of the solutions of the equation $x^2 + 2\sqrt{x^2 + 6x} = 24 - 6x$

A - 6

B - 2

C 2

D 6

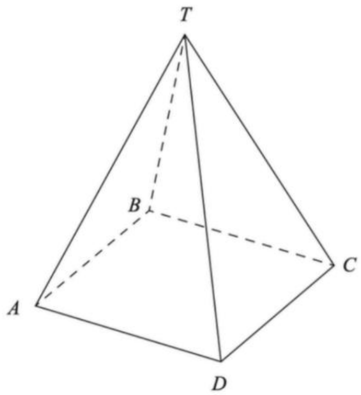
E 10

8. Given that $3^a = 16$ and $2^b = 27$, find the value of ab

A 3 B $\frac{7}{2}$ C 4 D $\frac{9}{2}$ E 12

16. A square based pyramid, with base ABCD, and vertex T has all edges of length 2m.

Find the shortest distance, in metres, along the outer surface of the prism from the midpoint of AB to the midpoint of CT.



A $\sqrt{3} - 1$

B 2

C $\sqrt{2} + 1$

D $\sqrt{4 + \sqrt{3}}$

E $2\sqrt{2}$

17. Three geometric transformations are defined as follows:

R is a reflection in the y-axis

S is a stretch parallel to the x-axis, scale factor 1/2

T is a translation by 3 units in the negative x direction

These three transformations are applied to the graph of $y = \sqrt{x}$ resulting in the graph of $y = \sqrt{3 - 2x}$
In which order were the transformations applied?

A R then S then T

B R then T then S

C S then R then T

D S then T then R

E T then S then R

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1. For how many values of the constant k does the following equation have only one real solution

$$kx^2 - (k-1)x + k = 0$$

A no values of k

B one value of k

C two values of k

D all values of k except $k = 1$

E all values of k

13. Consider the four options below about a particular statement:

A The statement is true if $x^2 < 1$

B The statement is true if and only if $x^2 < 1$

C The statement is true if $x^2 < 2$

D The statement is true if and only if $x^2 < 2$

Given that exactly one of these options is correct, which one is it?

19. Which of the following statements are true?

I **There exists** a real y such that **for all** real x, $y > x$

II **There exists** a real x such that **for all** real y, $x + y > xy$

III **For all** real x, **there exists** real y such that $x - y = xy^2 + 1$

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

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R2Drew2 TMUA MOCK TEST A PAPER 1
(4) D
(17) C
(20) A
R2Drew2 TMUA MOCK TEST A PAPER 2
(13) D
(17) E
(20) B
R2Drew2 TMUA MOCK TEST B PAPER 1
(2) C
(5) A
(8) E
(16) B
(17) E
R2Drew2 TMUA MOCK TEST B PAPER 2
(1) 3 roots
(13) A
(19) F

BEYOND HORIZON TMUA PAPER 1

1) For what values of x is $f(x)$ a valid function?

$$f(x) = \frac{1}{2\log(x^2 - 2x - 3)}$$

(A) $(0, \infty)$

(B) $(-\infty, -1) \quad x \neq 1 \pm \sqrt{5}$

(C) $(-\infty, -1) \cup (3, \infty) \quad x \neq 1 \pm \sqrt{5}$

(D) $(-\infty, -3) \cup (1, \infty) \quad x \neq 1 \pm \sqrt{5}$

(E) $f(x)$ is never valid

(F) $(-\infty, -1) \cup (3, \infty)$

4) What is the sum of the coefficients in $\left(\frac{2}{x} + \frac{3}{x^2} + 5x\right)^3$?

(A) 1000

(B) 750

(C) 125

(D) 343

(E) 100

(F) 729

8) There are seven greeting cards, each of a different colour, and seven envelopes of the same seven colours. The number of ways in which the cards can be put in the envelopes, so that exactly four of the cards go into the envelopes of the right colours, is

(A) $\binom{7}{3}$

(B) $2\binom{7}{3}$

(C) $(3!)\binom{4}{3}$

(D) $(3!)\binom{7}{3}\binom{4}{3}$

(E) $(3!)\binom{7}{3}$

(F) $(4!)\binom{7}{3}$

10) Let $p(x)$ be a continuous function which is positive for all x and

$$\int_2^3 p(x)\,dx = c\int_0^2 p\left(\frac{x+4}{2}\right)\,dx.$$

Then

(A) $c = 4$

(B) $c = -\frac{1}{4}$

(C) $c = \frac{1}{4}$

(D) $c = 2$

(E) $c = -\frac{1}{2}$

(F) $c = \frac{1}{2}$

12) Suppose $a < b$. The maximum value of the integral

$$\int_a^b \left(\frac{3}{4} - x - x^2\right)\,dx$$

over all possible values of a and b is

(A) $\frac{3}{4}$

(B) $\frac{4}{3}$

(C) $\frac{3}{2}$

(D) $\frac{2}{3}$

(E) $\frac{4}{5}$

(F) $\frac{5}{4}$

14) The mth term of an arithmetic progression is x and the nth term is y. What is the sum of the first $(m+n)$ terms?

(A) $\frac{m+n}{2}\left[(x+y) + \frac{x-y}{m-n}\right]$

(B) $\frac{m+n}{2}\left[(x-y) + \frac{x+y}{m-n}\right]$

(C) $\frac{1}{2}\left[\frac{x+y}{m+n} + \frac{x-y}{m-n}\right]$

(D) $\frac{1}{2}\left[\frac{x+y}{m+n} - \frac{x-y}{m-n}\right]$

(E) $\frac{m-n}{2}\left[(x-y) + \frac{x+y}{m-n}\right]$

(F) $\frac{m+n}{2}\left[(x+y) + \frac{x+y}{m-n}\right]$

15) The values of m for which $mx^2 - 6mx + 5m + 1 > 0$ for all real x is

(A) $0 < m < \frac{1}{4}$

(B) $0 \leq m < \frac{1}{8}$

(C) $m > 0$

(D) $0 \leq m < \frac{1}{4}$

(E) $m < 0$

(F) $m = 0$

16) The inequality $\sqrt{x+6} \geq x$ is satisfied for real x if and only if

(A) $-3 \leq x \leq 3$

(B) $-2 \leq x \leq 3$

(C) $-6 \leq x \leq 3$

(D) $0 \leq x \leq 6$

(E) $-6 < x < 6$

(F) $0 < x < 3$

17) The difference between the roots of the equation $6x^2 + \alpha x + 1 = 0$ is $\frac{1}{6}$. α is a positive number. The value of α is

(A) 2

(B) 3

(C) 4

(D) 5

(E) $\frac{8}{3}$

(F) 8

19) All the letters of the word PESSIMISTIC are to be arranged so that no two S's, no two I's, and S and I do not occur together. The number of such arrangements is

(A) 1800

(B) 5480

(C) 4800

(D) 1200

(E) 2400

(F) 1801

BEYOND HORIZON TMUA PAPER 2

8) Given that $\log_p x = \alpha$ and $\log_q x = \beta$, the value of $\log_{p/q} x$ equals:

(A) $\frac{\alpha\beta}{\beta-\alpha}$;

(B) $\frac{\beta-\alpha}{\alpha\beta}$;

(C) $\frac{\alpha-\beta}{\alpha\beta}$;

(D) $\frac{\alpha\beta}{\alpha-\beta}$.

10) Let S be the set of all numbers of the form $4^n - 3n - 1$, where $n = 1, 2, 3, \ldots$. Let T be the set of all numbers of the form $9(n-1)$, where $n = 1, 2, 3, \ldots$. Only one of the following statements is correct. Which one is it?

(A) Each number in S is also in T.

(B) Each number in T is also in S.

(C) Every number in S is in T and every number in T is in S.

(D) There are numbers in S which are not in T and there are numbers in T which are not in S.

15) When 15 is appended to a list of integers, the mean is increased by 2. When 1 is appended to the enlarged list, the mean of the enlarged list is decreased by 1. How many integers were in the original list?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

16) For how many integers n is $\frac{n}{20-n}$ the square of an integer?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 10

17) The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is:

(A) 11

(B) 12

(C) 13

(D) 14

(E) 15

BEYOND HORIZON TMUA PAPER 1

(1) C
(4) A
(8) B
(10) F
(12) B
(14) A
(15) D
(16) C
(17) D
(19) E

BEYOND HORIZON TMUA PAPER 2

(8) A
(10) A
(15) A
(16) D
(17) D

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