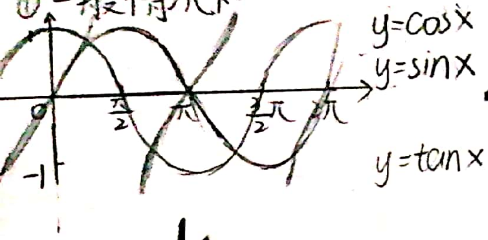
Trigonometry 三角函数

图形变化

① 一般情况下:



② 正常表达式

$y = a\,\sin/\cos(bx + c) + d$

- a → 振幅
- b → $T = \frac{2\pi}{|b|}$
- c → 左加右减
- d → 上下

attention:

$bx+c$ 与 $b(x+\frac{c}{b})$

解法：5点法 or
先算a、b、d，再代入b=0
求c

③ 反函数或解题

反函数：one-one function，
注意domain

解题：一般来说解不唯一
注意画图分析

④ 奇变偶不变，符号看象限

$\sin(a+k\cdot\frac{\pi}{2})$:
- $k\%2==1$，|原式| $=\cos\alpha$
- $k\%2==0$，|原式| $=\sin\alpha$

$\cos(a+k\cdot\frac{\pi}{2})$:
- $k\%2==1$，|原式| $=\sin\alpha$
- $k\%2==0$，|原式| $=\cos\alpha$

其中 $k\in Z$

☆ 可看作为"单位圆の妙用"的另一解释

恒等式

$\cot\alpha = \frac{1}{\tan\alpha}$

$\sec\alpha = \frac{1}{\cos\alpha}$

$\csc\alpha = \frac{1}{\sin\alpha}$

$1+\tan^2\alpha = \sec^2\alpha$

$1+\cot^2\alpha = \csc^2\alpha$

$\sin^2\alpha+\cos^2\alpha = 1$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$\cos^2\alpha = \frac{1+\cos2\alpha}{2}$

$\sin^2\alpha = \frac{1-\cos2\alpha}{2}$

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$

$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha\pm\beta) = \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

$\sin2\alpha = 2\sin\alpha\cos\alpha$

$\cos2\alpha = \cos^2\alpha - \sin^2\alpha$

$\tan2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\sin3\alpha = 3\sin\alpha - 4\sin^3\alpha$ 三山无可令

$\cos3\alpha = 4\cos^3\alpha - 3\cos\alpha$ 司令无山带

$\sin\alpha\pm\cos\alpha = \sqrt{2}(\sin\alpha\pm\frac{\pi}{4})$

$A\sin\alpha\pm B\cos\alpha = \sqrt{A^2+B^2}\,\sin(\alpha\pm\beta)$ 辅助角公式

for $\sqrt{A^2+B^2}$, B, A, β

$\sin(-\alpha) = -\sin\alpha$

$\cos(-\alpha) = \cos\alpha$

$\tan(-\alpha) = -\tan\alpha$

$\sin(\frac{\pi}{2}-\alpha) = \cos\alpha$

$\cos(\frac{\pi}{2}-\alpha) = \sin\alpha$

$\tan(\frac{\pi}{2}-\alpha) = \cot\alpha$

$\sin(\pi-\alpha) = \sin\alpha$

$\cos(\pi-\alpha) = -\cos\alpha$

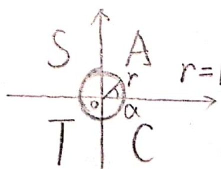
$\tan(\pi-\alpha) = -\tan\alpha$

互余都相反

互补sin不变

相反cos不变

单位圆の妙用

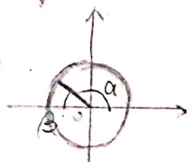


r=1.

$\sin\alpha = y$

$\cos\alpha = x$

$\tan x = \frac{y}{x}$



☆ 求任何一角度sin/cos值，只需写作"符号+与x轴夹锐角的三角函数"

易考定值

☆

$\sin15^\circ = \cos75^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\sin75^\circ = \cos15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\tan15^\circ = 2-\sqrt{3}$

$\tan75^\circ = 2+\sqrt{3}$

$\tan22.5^\circ = \sqrt{2}-1$

$\tan67.5^\circ = \sqrt{2}+1$

$\sin18^\circ = \cos72^\circ = \frac{\sqrt{5}-1}{4}$

哼哼哼太难了啊啊啊啊!

首

From 杨冠中 eCFdPB.

Differentiation VS. Integration

Differentiation

$\frac{d}{dx}[c]=0$

$\frac{d}{dx}[x^n]=nx^{n-1}$

$\frac{d}{dx}[e^x]=e^x$

$\frac{d}{dx}[a^{kx}]=k(\ln a)^{kx}$

$\frac{d}{dx}[f(x)\pm g(x)]=\frac{d}{dx}[f(x)]\pm\frac{d}{dx}[g(x)]$

$\frac{d}{dx}[f g(x)]=\frac{d}{dx}[f(g(x))]\cdot\frac{d}{dx}[g(x)]$

$\frac{d}{dx}[f(x)g(x)]=\frac{d}{dx}[f(x)]g(x)+\frac{d}{dx}[g(x)]f(x)$

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{\frac{d}{dx}[f(x)]g(x)-\frac{d}{dx}[g(x)]f(x)}{g^2(x)}$

$\frac{d}{dx}[f(x)g(x)u(x)]=\frac{d}{dx}[f(x)]g(x)u(x)+\frac{d}{dx}[g(x)]f(x)u(x)+\frac{d}{dx}[u(x)]f(x)g(x)$

$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$

$\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$

$\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1}x)=\frac{1}{x^2+1}$

$\frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\sin x]=\cos x$

$\frac{d}{dx}[\cos x]=-\sin x$

$\frac{d}{dx}[\tan x]=\sec^2 x$

$\frac{d}{dx}[\sec x]=\sec x\tan x$

$\frac{d}{dx}[\cot x]=-\csc^2 x$

$\frac{d}{dx}[\csc x]=-\csc x\cot x$

Integration

$\int 0\,dx=C$ $\int kf(x)dx=k\int f(x)dx$

* 多种写法

$\int x^n dx=\frac{x^{n+1}}{n+1}+C\ (n\neq 1)$

$\int \frac{1}{x}dx=\ln|x|+C$ * 注意绝对值

$\int e^x dx=e^x+C$

$\int e^{ax+b}dx=\frac{1}{a}e^{ax+b}+C$

$\int a^{kx}dx=\frac{a^{kx}}{k\ln a}+C$

$\int\frac{1}{ax+b}dx=\frac{1}{a}\ln|ax+b|+C$

$\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\frac{x}{a}+C$

$\int\frac{1}{x^2+a^2}dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$

$\int\cos x\,dx=\sin x+C$

$\int\sin x\,dx=-\cos x+C$

$\int\sec^2 x\,dx=\tan x+C$

$\int\sec x\tan x\,dx=\sec x+C$

$\int\csc^2 x\,dx=-\cot x+C$

$\int\csc x\cot x\,dx=-\csc x+C$

$\int u(x)\frac{d}{dx}v(x)dx=u(x)v(x)-\int v(x)\frac{d}{dx}u(x)dx$

Log Inverse Polyno(多项式) Expo(e^x) Trig

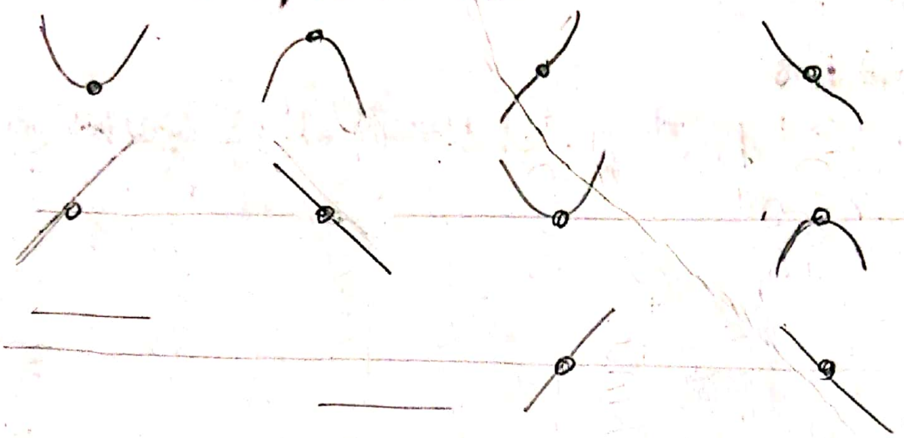
SOMETHING SPECIAL

①

原函数

一阶导 $y=0$

二阶导 $y=0$

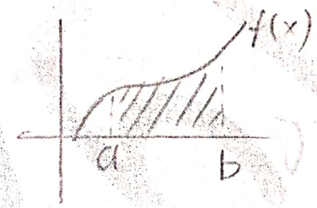


② 其余几种分解积分方法

1. 三角换元 当分母为 $\sqrt{x^2-a^2}$ 时
2. 因式分解 $\frac{c}{AB} = \frac{c_1}{A} \pm \frac{c_2}{B}$

③

求面积 → 定积分



$$A=\int_a^b f(x)\,dx$$

求体积：

$$V=\pi\int_a^b f^2(x)\,dx$$

求差值 → 上减下

AL MATH

一、绝对值(检验!)

① ±式子
平方(检验根)
归到两边

② $f(x)=(a_1x-b_1)(a_2x-b_2)(a_3x-b_3)\cdots$

$$\Updownarrow$$

$$f\left(\frac{b_1}{a_1}\right)=f\left(\frac{b_2}{a_2}\right)=f\left(\frac{b_3}{a_3}\right)=\cdots=0$$

③ a_n的positive & negative factors中有式子の解

④ $P(x)$除以$(ax-b)$余$P\left(\frac{b}{a}\right)$

二、log(检验)

① $\log_a b = c$ b为positive!

② x在指数上 用log弄下来

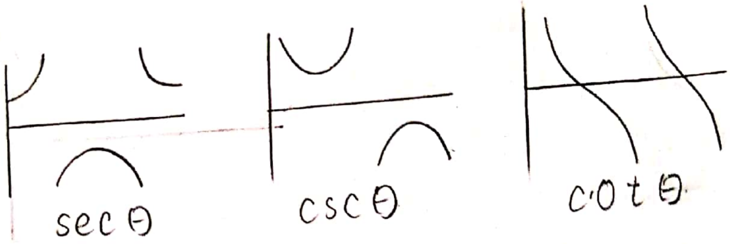
③ $y=ae^{bx} \iff \ln y = bx+\ln a$

数学狗都不学!

三、三角函数

① $1+\tan^2\theta=\sec^2\theta$
$1+\cot^2\theta=\csc^2\theta$

② $\cos\theta=\frac{1}{\sec\theta}$
$\sin\theta=\frac{1}{\csc\theta}$



③ $\sin 2\theta=2\sin\theta\cos\theta$
$\cos 2\theta=\cos^2\theta-\sin^2\theta=1-2\sin^2\theta=2\cos^2\theta-1$
$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$
$\sin^2\theta=\frac{1-\cos 2\theta}{2}$
$\cos^2\theta=\frac{1+\cos 2\theta}{2}$
$\tan^2\theta=1-\frac{2\tan\theta}{\tan 2\theta}$

④ $(\sin\theta)'=\cos\theta$ $(\csc\theta)'=-\csc\theta\cot\theta$
$(\cos\theta)'=-\sin\theta$ $(\sec\theta)'=\sec\theta\tan\theta$
$(\tan\theta)'=\sec^2\theta$ $(\cot\theta)'=-\csc^2\theta$

$\int\sin\theta\,d\theta=-\cos\theta+C$ $\int\csc\theta\cot\theta\,d\theta=-\csc\theta+$
$\int\cos\theta\,d\theta=\sin\theta+C$ $\int\sec\theta\tan\theta\,d\theta=\sec\theta+c$
$\int\sec^2\theta\,d\theta=\tan\theta+C$ $\int\csc^2\theta\,d\theta=-\cot\theta$

$(\tan^{-1}a\theta)'=\frac{a}{a^2\theta^2+1}$
$(\sin^{-1}a\theta)'=\frac{a}{\sqrt{1-a^2\theta^2}}$, $\int\frac{1}{\sqrt{a^2-\theta^2}}d\theta=\sin^{-1}\frac{\theta}{a}+c$
$(\cos^{-1}a\theta)'=-\frac{a}{\sqrt{1-a^2\theta^2}}$ $\int\frac{1}{\theta^2+a^2}d\theta=\frac{1}{a}\tan^{-1}\frac{\theta}{a}+c$

注:① $\square^2\theta$ 与 $\square 2\theta$ 化为 $\square\theta$
② $\tan\theta$换$\frac{\sin\theta}{\cos\theta}$, $\frac{1}{\square}$换$\square$
× try to use double-angle formula

四、积分

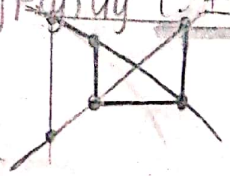
① $\int\frac{1}{x}dx=\ln|x|+C$ 加绝对值

② $\int_a^b f(x)\approx\frac{h}{2}[y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n]$ where $h=\frac{b-a}{n}$

③ $\int uv'dx=uv-\int u'vdx$
选u为u顺序: Log Inverse Polyno(多项式) Expo(e^x) Trig

④ $f(y)g(x)=\frac{dy}{dx}\Rightarrow g(x)=F(y)\frac{dy}{dx}\Rightarrow\int g(x)dx=\int F(y)dy$ (注意$y=0$) (general->particular)

五、迭算



将式子化为$x=f(x)$, 用Ans一遍遍找近似值
let $x_{n+1}=f(x_n)$, work out $f(x_{n+1})$; if it is accurate enough, $x=x_{n+1}$

六、拆分

① $\frac{px+q}{(ax+b)(cx+d)} \equiv \frac{A}{ax+b} + \frac{B}{cx+d}$

$\frac{px+q}{(ax+b)^2} \equiv \frac{A}{ax+b} + \frac{B}{(ax+b)^2}$

$\frac{px+q}{(ax+b)(cx^2+d)} \equiv \frac{A}{ax+b} + \frac{Bx+C}{cx^2+d}$

分子次高于分母 先长除再拆分

② $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$

where n is rational and $|x|<1$ (通过这个求ax中x的取值)

if $(a+x)^n$, turn it into $a^n(1+\frac{x}{a})^n$, where $|\frac{x}{a}|<1$

七、向量

① $\overrightarrow{OA} = a$, $\overrightarrow{OB} = b \Rightarrow \overrightarrow{AB} = b-a$

② $a = a_1 i + a_2 j + a_3 k \Rightarrow |a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

③ two vectors are parallel if one is a scalar multiple of the other

④ $a = a_1 i + a_2 j + a_3 k$, $b = b_1 i + b_2 j + b_3 k$

⇓

$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 = |a||b|\cos\theta$ (用这个式子求θ、证垂直(LHS=0))

⑤ vector equation: $r = a + tb = a + t(c-a)$

⑥ vectors: parallel / intersect / skew (既不相交也不平行) 检验交点是否存在

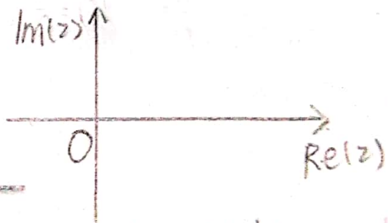
⑦ unit displacement vector: 除以r

⑧ P为vector外一点, 求P到向量l的最短距离: 1. 设N, 使NP⊥l, 用equation表示N
2. 写出 $\overrightarrow{ON}$ 与 $\overrightarrow{OP}$, 求出 $\overrightarrow{NP}$
3. $\overrightarrow{NP} \cdot l_1 = 0$

⑨ $ax+by+cz=d$
↓
normal vector: $ai+bj+ck$

八、虚数

① cartesian form: $z = x+iy$
modulus-argument form: $z = r(\cos\theta + i\sin\theta)$ where $r = |z|$ and $\theta = \arg(z)$
exponential form: $z = re^{i\theta}$



① if $z_1 = a+bi$, $z_2 = c+di$

$z_1 z_2 = (ac-bd) + (ad+bc)i$

$\frac{z_1}{z_2} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$ (上下同乘 $c-di$)

$\tan\theta = \frac{y}{x}$ $(-\pi \leq \theta \leq \pi)$

$|z_1 z_2| = |z_1||z_2|$

$\arg(z_1 z_2) = \arg z_1 + \arg z_2$

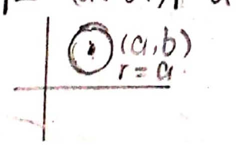
⑥ z^*与z共轭复数: $a \pm bi$

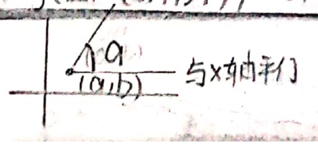
③ 二次方方程: 二实 or 二虚
三次方方程: 三实 or 一实二虚
四次方方程: 四实 or 二实二虚 or 四虚

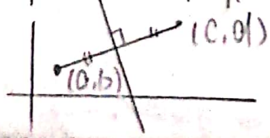
$a+bi$与$a-bi$同时出现
$x^3=1$, $x_1 = 1$, $x_2 = \frac{-1 \pm i\sqrt{3}}{2}$

④ 虚数的根: 列式解

⑤ $|z-(a+bi)| = a$ $\quad$ $\arg(z-(a+bi)) = a$ ※ $\quad$ $|z-(a+bi)| = |z-(c+di)|$







格式必须一模一样

一. stem-and-leaf diagram

x | xx xx (xx)
x | xx (xx)
x | xx xx xx (x)

key: x|xx represents xxx

xx		xx	
(x) xx	x	xxxx	(x)
(x) xxx	x	x	(x)
(x) x	x	xx	(x)

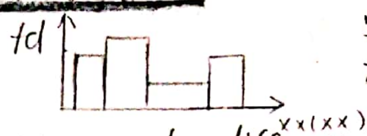
key x|x|x represents

二. class boundary

① continuous 才用出去

② 实际问题综合考虑（有无小数 e.g.）

三. histogram



写出对应的fd

fd最大的为modal class

四. cumulative frequency curve

写出对应坐标

x坐标取class boundary

五. compare the different data representations

histogram / cumulative frequency curve:
large amount + continuous data
(no raw data + no small amount)

stem-and-leaf diagram
small amount (no large amount, no tendency)

* 离散形, class boundary +1!

六. median & mean & mode + further calculations

	ad	disad
mode	unlikely to be affected by extreme value useful to know the most popular one can be used for all sets of qualitative data	ignore most values rarely used in further calculations
mean	take all values in account frequently used in further calculations the most commonly understood average can be used to find the sum of the data values	cannot be found unless all values are known likely to be affected by extreme values
median	can be found without knowing all values relatively affected by extreme values	ignore most values

* using IQR & Q2 find larger & more spread out

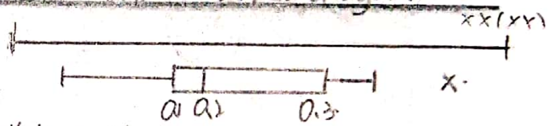
$$\begin{cases} \bar{x} = \overline{x-b} + b \\ \bar{x} = \frac{1}{a}(\overline{ax-b} + b) \end{cases}$$

$$\begin{cases} SD(x) = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \\ SD(x) = SD(x-b) \\ SD(x) = \frac{1}{a} SD(ax-b) \end{cases}$$

七. percentile

n th percentile：从小至大的n%

八. box-and-whisker diagram



考虑 outlier

九. skewed

symmetrical

negatively skewed

positively skewed

十. mutually exclusive events

if $P(A\cap B)=0$, A and B are mutually exclusive events.

for them, $P(A\cup B)=P(A)+P(B)$, for else, thinking of $P(A\cap B)$

十一. permutation 排列

① $^nP_n = n!$

② for the number of permutations of n objects, of which p are of one type, q are of another type…: $\frac{^nP_n}{p!\times q!\times r!\times\cdots}$ where $n=p+q+r+\cdots$

③ $^nC_r \cdot {}^rP_r = {}^nP_r$

$^nC_r = \frac{n!}{r!(n-r)!}$

$^nC_r \leq {}^nP_r$

十一. independent events & dependent events

① if either can occur without being affected by the occurrence of the other, they're independent events

for them, $P(A\cap B) = P(A)\times P(B)$

if $P(X|Y) = P(X|Y')$, then they're independent events.

② for independent events $P(A\cap B) = P(A)\times P(B|A)$

for all events, $P(A|B) = \frac{P(B\cap A)}{P(B)}$

十二. discrete random variables & probability distributes

① discrete random variable: occur by chance 有不同概率

② probability distribute: display of all its possible values and their corresponding probabilities 有相同概率

③ for discrete random variable $\begin{cases} E(x) = \bar{x} = \Sigma xp \\ Var(x) = \Sigma x^2 p - [E(x)]^2 \\ \Sigma p = 1 \end{cases}$

十三. binomial distribution & geometric distribution

① $\begin{cases}\text{binomial distribution 二项分布, n次实验中成功几次} \\ \text{geometric distribution 几何分布, n次成功需几次实验}\end{cases}$

* 考虑 $P(X=0)$ 时的情况

② if $X\sim B(n,p)$, the probability of r successes is P_r(成功几次) $= {}^nC_r\, p^r(1-p)^{n-r}$

where there are n repeated independent trials, two possible outcomes, the possibility of success in each trial is p, a costant

mean: $\mu = np$ $(\mu = E(x))$

variance: $\sigma^2 = np(1-p)$ $(\sigma^2 = Var(x))$

$1-p$ (or q) $= \frac{npq}{np} = \frac{Var(x)}{E(x)}$

f. $X \sim Geo(p)$. the probability that the first success occurs on the nth trial: p_r (第几次成功) $p(1-p)^{r-1}$

where the repeated trials are idependent. there are 2 possible outcomes the probability of success in each trial p, is constant

$P(X \le r) = P$(success on one of the first r trials)
$= 1 - P$(failure on the first r trials)

$P(X > r) = P$(first success after the rth trials)
$= P$(failure on the first r trials)

turn $P(X \ge r)$ into $P(X > r+1)$, $P(X < r)$ into $P(X \le r-1)$

mean $\mu = \frac{1}{p}$ variance $\sigma^2 = \frac{p^2}{q}$

mode: $P(X=1)$, cuz $p > p(1-p) > p(1-p)^2 > \cdots$

$Var(x) = E((X - E(x))^2)$
$= E(X^2 - 2XE(x) + E(x)^2)$
$= E(X^2) - 2E(x)E(E(x)) + E(E(x)^2)$
$= E(X^2) - 2E(x)E(x) + E(x)^2$
$= E(X^2) - E(x)^2$

binomial
- n repeated independent trials
- n is finite
- two possible outcomes
- p is constant

geometry
- repeated trials are independent
- can be infinite
- two possible outcomes
- p is constant

$\hat{p}$ 代表sample中成功概率

$E(\hat{p}) = E(\frac{X}{n}) = \frac{1}{n}E(X) = \frac{1}{n}np = p$

$Var(\hat{p}) = Var(\frac{X}{n}) = \frac{1}{n^2}Var(X) = \frac{1}{n^2} \cdot npq = \frac{pq}{n}$

注意 $\hat{p}$!

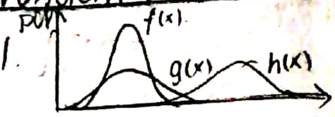
$\therefore$ 转化为 $N(p, \frac{pq}{n})$

CI of proportion:

$(p - k\sqrt{\frac{pq}{n}}, p + k\sqrt{\frac{pq}{n}})$

$SD(\hat{p})$

十四 continuous random variables



① for normal curve: mean = median = mode
peak at mean (μ)
probability decreases when $|x - \mu|$ increases
standard deviation (σ) increases mean more spread out
e.g. $\sigma_f < \sigma_g$ $\mu_g < \mu_h$

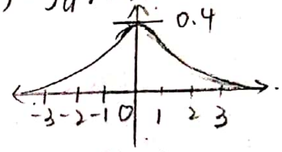
② if the random variable X is normally distributed with mean μ and variance σ^2, then its function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$$ for any real x

to discribe, we write $X \sim N(\mu, \sigma^2)$

so $P(a \le X \le b) = \int_a^b f(x)\,dx$

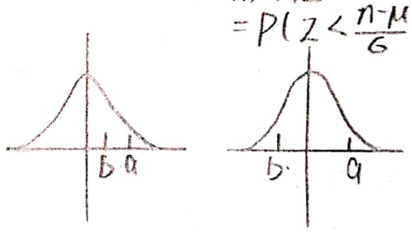
③ $Z \sim N(0,1)$
$\phi(z)$ means $P(Z \le z)$



for a positive value b, $\phi b = P(z \le b)$
for a negative value a, $\phi a = P(z \ge -a)$

④ turn $X \sim N(\mu, \sigma^2)$ into $Z \sim N(0,1)$:
$Z = \frac{X - \mu}{\sigma}$ ※将一般normal curve化为标准normal curve. $P(X < n) = P(Z < \frac{n-\mu}{\sigma})$

⑤

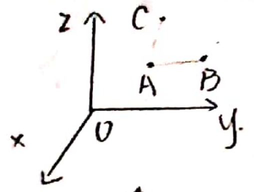


$P(b < z < a) = \phi(a) - \phi(b)$ $P(b < z < a) = \phi(a) + \phi(b) - 1$

※由P倒推μ或σ:
先判断a.b大于/小于mean, 再代数

⑥ 用normal估计binomial
首先要满足 $np, nq \ge 5$
然后用 $z = \frac{X - \mu}{\sigma}$ 求z.
(注意, 如X为discrete, 则需用其boundary计算) 代入求解.

九. 关于立体向量



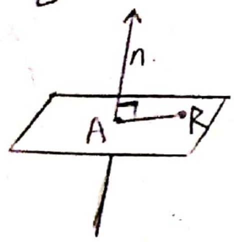
已知A、B、C，设R为平面上任意点$\begin{pmatrix}x\\y\\z\end{pmatrix}$

$\vec{OA}=a$, $\vec{OB}=b$, $\vec{OC}=c$, $\vec{OR}=r$.

A、B、C、R在同一平面

∴ $r=\vec{OA}+\lambda\vec{AB}+\mu\vec{BC}=a+\lambda(b-a)+\mu(c-a)=\begin{pmatrix}x\\y\\z\end{pmatrix}$

解方程即可



将direction设为$n=n_1i+n_2j+n_3k$，$\vec{OA}=a$，R为平面上任意点$\begin{pmatrix}x\\y\\z\end{pmatrix}$

$\vec{AR}\cdot n=0$

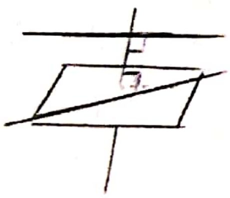
$(r-a)n=0$

$rn-an=0$

$\begin{pmatrix}x\\y\\z\end{pmatrix}\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix}=an \longrightarrow n_1x+n_2y+n_3z=d$

※此为法向量，求垂直平面

$\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix}$与$n_1x+n_2y+n_3z=d$垂直



线面关系：相交 → 求μ找交点（垂直 → 与法向量正比例）

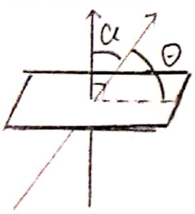
平行 → 与法向量垂直

在面上 → 证相交+平行

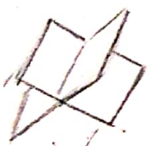
点面距离：法向量作direction，写出过点equation $y=\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}+\mu\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix}$

找交点，算距离

given $\dfrac{|n_1\alpha+n_2\beta+n_3\gamma-d|}{\sqrt{n_1^2+n_2^2+n_3^2}}$

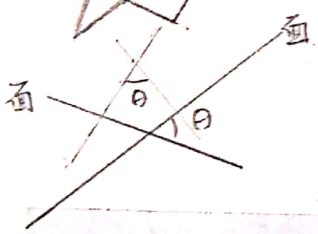


线面夹角：线与法向量夹角α，$\theta=\frac{\pi}{2}-\alpha$

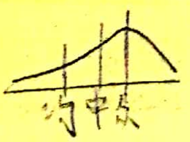


两面交线：① 将x、y、z分别为0求两点，然后算线

② 将y、z用x表示，得$x=\lambda$, $y=k_1\lambda+b_1$, $z=k_2\lambda+b_2$，则equation为$y=\begin{pmatrix}0\\b_1\\b_2\end{pmatrix}+\begin{pmatrix}\lambda\\k_1\lambda+b_1\\k_2\lambda+b_2\end{pmatrix}$

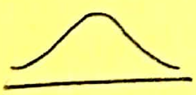


两面夹角 用法向量求θ



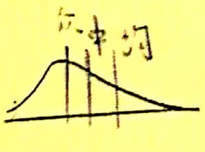
negatively skewed

mean < median



symmetric (roughly)

mean = median



positively skewed

mean > median

易错

读题! 带高级计算器!

P1

(factor)

① stretch with, translate by, reflect in ⟶ 代数、画图检查 ⑧ 面积/argu

② ±Ja保留看domain ⑦ 开根取正负. ⑨ range 画图、表格 $(\frac{a}{x+b})$

③ $\frac{m}{n}$, $n, m \in N^*$, $n>a$ ⟶ $0<\frac{m}{n}<\frac{m}{a}$

④ 三角函数画图! 不要随意舍根!

⑤ 同除a, $a \neq 0$

⑥ radian & degree.

范围看题目!

离散&连续? way or probability?

排列&组合? 重复&权重?

S1

一、排列组合

① next to each other/(not) all together ⟶ 读题、^mP_m中m取值, 注意not.

② A、B于左右固定 ⟶ A、B已固定, 算重复时无关两边A、B, 注意^mP_m中m取值.

③ A、B、C个东西中选x个, 其中每项至少选a、b、c个 ⟶ 必须分类讨论! 否则会重复! 考虑分类情况中的重复, 不可乱乘^mP_m. 画出所有情况来求和.

$P(A) = 1 - P(A')$

④ A去B不去, B去A不去 ⟶ 分别作讨论, 但想明白^mC_n的m、n的含义! 可(1−P')求

⑤ 插空法 ⟶ ^mP_n考虑m的取值. 有没有必要插空(相邻的东西为什么不能在一起?)

二、normal distribution

① $N(\mu, 6^2)$, 注意6与6^2 ② discrete要扩 ③ $6^2=npq$, $6^2=\frac{\sum x^2}{n}-\bar{x}^2$ ④检验: np, nq

三、combination 的 probability 用重复代表权重(全部重复)

gradient or equation?

x-coordinate or coordinate.

四、画图

① 离散型数据考虑扩 cf/df 图 boundary!

② box and whisker 的 outlier 画 x, 边缘画至 Q1−1.5IQR 与 Q3+1.5IQR

③ 离散求 IQR_{max} ④ stem-and-leave 注意 stem 与 key

P3

degree or radian?

① $\int \frac{1}{x}dx = \ln|x| + C$ ② 画图(圆&直) 用虚/实线, 交点一定要交上!

③ $\log a$, a必须positive (排除根) ④ 两线关系 ⟶ 方向、大小

⑤ 代成 $e^y - c \cdot e^x$ 与 $y = x + C$ ⟶ in terms of y

⑥ vector求angle ⟶ $\pi - \theta$? $\frac{\pi}{2} - \theta$? θ? 看准长度 ⑦ 点出 $\frac{dy}{dx} = \cdots$ ⑧ parallel to x-axis or y-axis?

⑨ 展开看正负 ⑩ trapezium rule 估大估小? 画图仔细想!

⑪ a^x/$\log_a x$ 趋势? ⑫ $\vec{AB}=k\vec{BC}$, $\vec{AB}=\frac{k}{k+1}\vec{AC}$

⑬ 画argu: < or >

⑭ $|z+k|<b$ and $\arg(z+k)<b$: 实部虚部? + or −?

⑮ distinct!

M

① 考虑外力做功损失能量 (energy method)

② 能量守恒? 系统/个体?

③ displacement 不能单纯求积分. 考虑 Δv、Δa、v、a 正负!

④ θ° clockwise/anticlockwise to positive/negative x/y-direction.

⑤ 求最高h: 先求V0时, 再用a求$s_{总}$: 注意a有无T, 算g, 是否自由上抛

滑轮 S、V、T 永相同

最后 h = xx + xx + s

单位统一 (最后换)

⑥ P = FV

⑦ 球于滑坡: N = G cos θ? 有无上方力?

S的亡灵:

① estimate 数据时只看 class mid-data 与 frequency

② 上一问条件不要代入下一问(Q7)

③ 排除法时, exceptation 考虑重复 EE E or E EE

全体目光向我看齐，我宣布个事！我是P23

parallelogram 平行四边形
rectangle 矩形
rhombus 菱形
trapezium 梯形

equilateral triangle
isosceles triangle

quadrilateral
pentagon
hexagon

物理简答 写错了就是井]?

- 水波 interference

① ensures the waves are coherent
the dippers are connected to the same vibrator

类似的，双逢

let the same beam of light pass through a single-slit in order to create coherent waves

② seeing the illuminated pattern more clearly

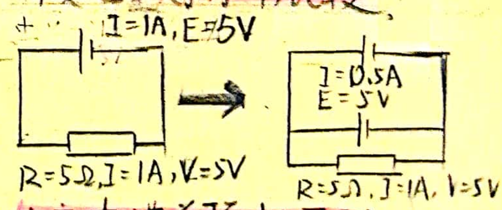
use strobe 频闪 "freeze"

- crust: 山峰 trough: 谷

- 电学题中 A 改变 $R=\frac{\rho L}{A}$, $I=Anvq$

⇓

一个变化量大于另一个，比较

- 

注意电池并联！(同压)

- work done 与 energy 相互转化
- node: 永远 destructive interference
antinode: 永远 constructive interference
- ionising 电离: α>β penetrate: β>α

archimedes