Mathematics Interview Questions

1. Differentiate x^x
2. Integrate $\cos^2(x)$ and $\cos^3(x)$.
3. What is the square root of i? (let i=exp(i*pi))
4. If I had a cube and six colours and painted each side a different colour, how many (different) ways could I paint the cube? What about if I had n colours instead of 6?
5. Prove that root 2 is irrational.
6. Integrate ln x.
7. Sketch the curve $(y^2-2)^2+(x^2-2)^2=2$. What does it look like?
8. 3 girls and 4 boys were standing in a circle. What is the probability that two girls are together but one is not with them?
9. Prove that 1+1/2+1/3+...+1/1000<10
10. Is there such number N that 7 divided $N^2=3$?
11. What is the integral of $x^2 \cos^3(x)$?
12. How many squares can be made from a grid of ten by ten dots (ignore diagonal squares)?
13. Integrate tan x.
14. Pascal's triangle (prove that every number in the triangle is the sum of the two above it)
15. Integrate 1/(1-lnx)
16. sketch x^x (x=a+ib)
17. prove 4^n - 1 is a multiple of 3 (let 4=3+1 and expand using binomial theorem)
18. How many ways there are of getting from one vertex of a cube to the opposite vertex without going over the same edge twice?
19. What shape there would be if the cube was cut in half from diagonally opposite vertices?
20. Draw x ln(x).
21. Integrate and differentiate xln(x).
22. Draw sin(1/X).
23. Differentiate $x^{(x\wedge x)}$
24. What do you know about triangles?
25. Find a series of consecutive integers such that the sum of the series is a power of 2.
26. Prove Ptolemy's Theorem.
27. Find roots of the equation mx=sin x considering different values of m.
28. Integrate $|\sin^n(x)+\cos^n(x)|$ between 0 and 2 & (pi) for cases n=1,2
29. $x^2 + y^2 = z^2$. Prove xyz is a multiple of 60
30. Two people are playing a game which involves taking it in turns to eat chillies. There are 5 mild chillies and 1 hot chilli. Assuming the game is over when the hot chilli is eaten (and that I don't like hot chillies), is it a disadvantage to go first? What is the probability that I will eat the chilli if I go first? How about if there are 6 mild and 2 hot?
31. $kx^4=x^3-x$ Find the real roots when k=0. Sketch the LHS and the RHS when k is small and then when k is large, and find approximation of the +ve root when k<<1. (let x=x_0+1, where x_0<<1, solve for x_0).
32. Sketch $f(x) = (x - R(x))^2$, where R(x) is x rounded up or down in the usual way. then sketch g(x) = f(1/x)
33. (a+b)/2 is an integer, is the relationship transitive? (a+b)/3?
34. Differentiate 1/1+(1/1+(1/1+1/(1 + x))))
35. Sketch graph of 1/x, $1/x^2$, $1/(1+x^2)$
36. Integrate $1/(1+x^2)$
37. Integrate $e^x x^2$ between limits of 1 and 0. Draw that graph.
38. Integrate x^{-2} between limits of 1 and -1. Draw the graph. Why is it -2 and not infinity, as it appears to be on the graph?
39. Write down 3 digits, and then write the number again next to itself, eg: 145145. Why is it divisible by 13? (abcabc=1001*abc)

40. You are given a triangle with a fixed perimeter but you want to maximise the area. What shape will it be? Prove it. (Herons formulate, A^2=p(p−a)(p−b)(p−c), 2p = a+b+c), sub in c=2p-a-b, then set partial derivatives equal to zero. Or assume otherwise, that a=/=b maximises A^2, then show setting a,b-->(a+b/2) in the formula gives greater than A^2, contradiction)
41. Next you are given a quadrilateral with fixed perimeter and you want to maximise the area. What shape will it be? Prove it.
42. Integrate $(1)/(x+x^3)$, $(1)/(1+x^3)$, $(1)/(1+x^n)$
43. How many 0's are in 100!
44. Prove that the angle at the centre of a circle is twice that at the circumference.
45. How many ways are there in which you can colour three equal portions of a disc?
46. Integrate $1/(9 + x^2)$
47. Draw $y=e^x$, then draw y=kx, then draw a graph of the numbers of solutions of x against x for e^x=kx, and then find the value of k where there is only 1 solution.
48. Rubik's cube and held it by two diagonally opposite vertices and rotated it till it reached the same position, by how many degrees did it take a turn?
49. Solve $a^b=b^a$ for all real a and b.
50. There is a game with 2 players (A&B) who take turns to roll a die and have to roll a six to win. What is the probability of person A winning?
51. Sketch $y=x^3$ and $y=x^5$ on the same axis.
52. What the 2 sides of a rectangle (a and b) would be to maximise the area if a+b=2C (where C is a constant).
53. Can 1000003 be written as the sum of 2 square numbers?
54. Show that when you square an odd number, you always get one more than a multiple of 8.
55. Prove that 1 + 1/2 + 1/3 + 1/4 + ... equals infinity
56. Prove that for n E Z ,n>2, $n^{(n+1)}>(n+1)^n$
57. Prove that sqrt(3) is irrational (or sqrt(2))
58. What are the possible unit digits for perfect squares?
59. What are the possible remainders when a cube is divided by 9?
60. Prove that 801,279,386,104 can't be written as the sum of 3 cubes
61. Sketch y=ln(x)/x and find the maximum.
62. What's the probability of flipping n consecutive heads on a fair coin? What about an even number of consecutive heads?
63. Two trains starting 30km apart and travelling towards each other. They meet after 20 mins. If the faster train chases the slower train (starting 30km apart) they meet after 50mins. How fast are the trains moving? (easy)
64. A 10 digit number is made up of only 5s and 0s. It's also divisible by 9. How many possibilities are there for the number?
65. There is a set of numbers whose sum is equal to the sum of the elements squared. What's bigger: the sum of the cubes or the sum of the fourth powers?
66. Draw $e^{(-x^2)}$
67. Draw $\cos^{(x^2)}$
68. What are the last two digits of the number which is formed by multiplying all the odd numbers from 1 to 1000000?
69. What are the possible values of a square number mod 5? Prove that 1!+ 2!+ 3!+... has no square values for n>3. (expand to 1!+2!+3!+4!+5!+.. everything from 5! onwards is divisible by 5).
70. How many zeros in 365!
71. Integrate $x \sin^2 x$
72. Draw e^x , In x, y=x what does show you. As x tends infinity, what does lnx/x tend to?
73. Define the term 'prime number'
74. Find method to find if a number is prime.

75. Prove for $a^2 + b^2 = c^2$ a and b can't both be odd.
76. What are the conditions for which a cubic equation has two, one or no solutions?
77. What is the area between two circles, radius one, that go through each other's centres?
78. If every term in a sequence S is defined by the sum of every item before it, give a formula for the nth term
79. Is 0.9 recurring = 1? Why? Prove it
80. Why are there no Pythagorean triples in which both x and y are odd?
81. draw a graph of sinx, $\sin^2 x$, $\sin^3 x$
82. prove infinity of primes, prove infinity of primes of form 4n+1
83. differentiate $\cos^3(x)$
84. Show $(x-a)^2 - (x-b)^2 = 0$ has no real roots if a does not equal b in as many ways as you can.
85. Hence show: i) $(x-a)^3 + (x-b)^3 = 0$ has 1 real root ii) $(x-a)^4 + (x-b)^4 = 0$ has no real roots iii) $(x-a)^4 + (x-b)^4 = (b-a)^4$ has 2 real roots
86. Find the values of all the derivatives of $e^{(-1/x^2)}$ at x=0
87. Show that n^5-n^3 is divisible by 12. (=n^3(n+1)(n-1))
88. If I have a chance p of winning a point in tennis, what's the chance of winning a game
89. Explain what integration is.
90. If n is a perfect square and its second last digit is 7, what are the possibilities for the last digit of n and can you show this will always be the case?
91. How many subsets can you form from a set of n numbers?
92. Prove that (a+b)/2 > sq.root of ab where a>0, b>0 and a does not equal b ie prove that arithmetic mean > geometric mean
93. What is 0^0 (i.e is it 0 or1), and solve it by drawing x^x
94. If f(x+y)=f(x)f(y) show that f(0) = 1,
95. Suggest prime factors of 612612503503
96. How many faces are there on an icosahedron
97. integrate 1/(1+sin x)
98. What is the greatest value of n for which 20 factorial is divisible by 2^n?
99. Prove that the product of four consecutive integers is divisible by 24.
100. Where is i^i in the complex plane?
101. What is bigger e^pi or pi^e?
102. Prove that the square of any odd number minus one equals a multiple of 8.
103. Sketch x^3 and x^5.
104. How many zeros are there at the end of 100!
105. How many numbers satisfy conditions:
    a. They have 3 digits.
    b. All digits are even.
    c. Digits are distinct.
106. If a+1/a is an integer, prove that so is a^2+1/a^2.
107. Sketch y=x^x.
108. Prove there are infinitely many prime numbers.
109. Sketch |x|+|y|=1.