Oxbridge Interview Questions for Mathematics and Sciences

These are questions from real Oxbridge interviews. Questions in no particular order. Some might need a bit of background reading.

1. Differentiate $\dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x+1}}}$ w.r.t. x.
2. Which is the quicker round trip by plane: no wind or a constant head wind (in one direction)?
3. Practise graphing any $y = f(x)$ and then $y = 1/f(x)$.
4. $\displaystyle\int_0^1 x^2 e^x \; dx$
5. Prove that of x is odd then 8 divides $x^2 - 1$.
6. Find $\dfrac{d}{dx}(x^x)$.
7. Repeat with x^{x^x} etc.
8. Where does $y = x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}}$ exist (i.e. what is its domain)?
9. Explain the 'Monty Hall' problem. [3 doors, 2 goats, 1 car prize.]
10. Think of a 3-digit number, e.g. 145, write a copy next to it, i.e. 145145. Explain why 13 divides this number always.
11. Investigate $x_{n+1} = kx_n(1-x_n)$ for different values of k. Investigate limiting behaviour, if any. (Use a computer.)
12. Explain why the length sum of the diagonals of any quadrilateral is less than its perimeter.
13. Can you show that any $\sqrt{p}$ is irrational for p prime?
14. How many 0's are there at the end of 100! ?
15. Using three colours, in how many different ways can you colour a disc split into three equal portions?
16. Show graphically how many solutions there are to $e^x = kx$ for different values of k.
17. Prove that $n^2 \equiv (n+7)^2 \mod 7$.
18. If $u_n \to 0$ as $n \to \infty$, does $\sum u_n$ converge? Proof or counterexample.
19. Solve $a^b = b^a$ for $a, b \in \mathbb{R}$. What does this have to do with $y = \dfrac{\ln x}{x}$?

20. Two players A and B roll a die. The first to rall a eix wins. Find P(A wins). What about 3 players A, B and C with P(A wins)?

21. Sketch $y = x^3$ and $y = x^5$ on the same axis. Also for $y = x^{103}$ and $y = x^{105}$.

22. Maximise the area of a rectangle $a \times b$ if $a + b = 2c$, where c is a constant.

23. What is the remainder when you divide a perfect number by 4?

24. Prove $n^{n+1} > (n+1)^n$ for $n \geq 3$.

25. What are the possible unit digits for perfect squares?

26. What are the possible remainders on dividing a perfect cube by 9?

27. What is the maximum of $y = \dfrac{\ln x}{x}$?

28. A ten digit number is made up only of 5s and 0s. It is divisible by 9. How many possibilities are there for the number?

29. Draw the graph of $y = e^{-x^2}$.

30. Find $\displaystyle\int \frac{dx}{x \ln x}$.

31. Graph $y = \cos(x^2)$.

32. Divide a cake, which is a cube, into 7 equal portions with same volume and surface area.

33. Find the last two digits of the number formed by multiplying all the odd numbers from 1 to 10^6.

34. Show that $1! + 2! + 3! + \cdots + n!$ is never square for $n > 3$.

35. Graph $y = x^x$, $y = x^{x^x}$ etc.

36. How many zeroes are there at the end of 365! ?

37. Find $\displaystyle\int x \sin(x^2)\ dx$.

38. Graph $y = \max(1, x)$ for $0 \leq x \leq 3$. What is the area under the graph?

39. How far do you need to look for a factor before deciding a number is prime?

40. Is it possible for $a, b \in \mathbb{Z}$ to be odd in $a^2 + b^2 = c^2$? Prove either way.

41. Graph $y = \sin\left(\dfrac{1}{x}\right)$.

42. Under what condition does a cubic equation have no, one, two or three solutions?

43. Graph $y = \sin x,\ \sin^2 x,\ \sin^3 x, \cdots$, any thoughts on $y = \sin^n x$.

44. Two unit circles pass through each other's centres. What is the area inside?

45. Explain why $0.\dot{9} = 1$.

46. Given certain starting value, define a sequence where each term is the sum of all the preceding terms. Investigate the n^{th} term.

47. Explain the problem and solution of the Bridges of Königsberg (Euler).

48. Prove there are an infinite number of primes (Euclid).

49. Prove there are an infinite nuber of primes of the form $p = 4n + 1$.

50. Graph $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$. What about $\dfrac{x^2}{4} + \dfrac{y^2}{9} + \dfrac{z^2}{49} = 1$ (in 3D)?

51. Find $\dfrac{d}{dx}\, e^{e^{e^x}}$.

52. Graph $y = \log x$, $y = \log\log x$, $y = \log\log\log x$ etc.

53. Show $(x-a)^2 - (x-b)^2 = 0$ has no real roots if $a \neq b$ in as many ways as you can.

    Hence, show that *i*, $(x-a)^3 + (x-b)^3 = 0$ has one real root.

    ii, $(x-a)^4 + (x-b)^4 = 0$ has no real roots.

    iii, $(x-a)^4 + (x-b)^4 = (b-a)^4$ has 2 real roots.

54. Find the value of the infinite continued fraction $[1, 1, 1, 1, \cdots]$.

55. What are $\left.\dfrac{d^n}{dx^n}\left(e^{-\frac{1}{x^2}}\right)\right|_{x=0}$ $\forall n \in \mathbb{N}$? What does this mean for its Maclaurin series?

56. Find the maximum value of $\dfrac{1}{6 + 3\sin x + 4\cos x}$.

57. Prove that 12 divides $n^5 - n^3$, what about 24?

58. Given $x^2 - 2x + 2$ has roots α and β find a quadratic with roots α^2 and β^2 *without* calculating α and β.

59. My probability of winning a point in tennis is p. What is probability I win a game?

60. A is a 2×2 matrix, investigate e^A.

61. n is a perfect square and its penultimate digit is 7. What is the last digit of n (more than one possibility)?

62. How many subsets are there of n numbers?

63. 50 people go to a party and shake hands with a random number of people, no one shakes the same person's hand twice. Is it possible no two people shake hands the same number of times?

64. If $f(x+y) = f(x)f(y)$ for all x and y, show that $f(0) = 1$.

65. Can 100003 be a sum of two perfect squares? Prove your assertion.

66. Look up Fermat's little theorem.

67. Look up Carmichael numbers.

68. What is the square root of i?

69. How many different ways can I colour a cube with six different colours (all used). What if there are n colours?

70. Graph $(y^2-2)^2+(x^2-2)^2=2$.

71. 3 girls and 4 boys standing in a circle. Find P(2 girls together and 1 someone else).

72. Does 7 divide n^2-3.

73. Prove $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{1000}<10$.

74. Prove that $\displaystyle\sum_{k=1}^{n}\frac{1}{k}$ is never an integer for $n>1$.

75. What about $\displaystyle\sum_{k=m}^{n}\frac{1}{k}$?

76. How many squares can you make on a grid of 10×10 dots (no diagonal squares)?

77. How many digits has 2^k?

78. Prove 3 divides 4^n-1.

79. In now many ways can you travel from one vertex of a cube to the opposite vertex without traversing the same edge twice?

80. What shapes do you get cutting through a cube with a plane in various ways?

81. Graph $y=x\log x$ and find $\displaystyle\int x\log x\;dx$.

82. How many triangle centres do you know?

83. Find a series of consecutive integers such that the sum is a power of 2?

84. At which points on a roller coaster are you most likely to fall off?

85. Look up Ptolemy's theorem and prove it.

86. Find roots of $mx=\sin x$ for different values of m.

87. Find $\displaystyle\int_0^{2\pi}|\sin^n x+\cos^n x|\;dx$ for $n=1,2,\cdots$.

88. If $x^2+y^2=z^2$, $x,y,z\in\mathbb{Z}$, prove that 60 divides xyz.

89. Two people are taking it in turn to eat chillies. There are 5 mild chillies and 1 hot chilli. Game over when hot chilli is eaten. Is it a disadvantage to go first? What if 6 mild and 2 hot?

90. Consider $kx^4=x^3-x$. What are the real roots when $k=0$? Sketch graphs when k is large and when k is small and approximate the roots in each case.

91. Sketch $y=f(x)=\big(x-R(x)\big)^2$, where $R(x)$ mean x rounded to the nearest integer. Now sketch $y=f(1/x)$.

92. Graph $y = \dfrac{1}{1+x^2}$.

Write down all the ways of permuting $1, 2, 3$. E.g. (12) means $1 \mapsto 2$, $2 \mapsto 1$ and $3 \mapsto 3$; (123) means $1 \mapsto 2$, $2 \mapsto 3$ and $3 \mapsto 1$. Look at what happens if you compose these permutations?

93. Graph $y = x^{1/100}$ and $y = x^{1/101}$.

94. Find $\displaystyle\int \frac{1}{1+\sin x}\, dx$.

95. What is the greatest calue of n for which 2^n divides 20! ?

96. Show that 24 divides the product of any 4 consecutive integers.

97. What is i^i?

98. You arrive at Rochester with £x in your bank account. Each time you visit the bank you withdraw half of what's left, plus £1 to donate to the appeal to erect a statue of Brian in front of the cathedral. How much is left after i, 1 ii, 2 iii, 3 and iv, n visits to the bank?

99. Prove that $1^5 + 2^5 + \cdots + n^5 = \frac{1}{12}n^2(n+1)^2(2n^2+2n-1)$.

100. Prove that $\displaystyle\sum_{k=0}^{n} \binom{n}{k} = 2^n$, $\displaystyle\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$ and $\displaystyle\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$.